Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

B.E. Sem-I Remedial Examination March / April 2010

Subject code: 110008 **Subject Name: Mathematics - I**

Date: 08 / 04 / 2010 **Time: 12.00 Noon – 03.00 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1

(a)

1. Find the value of $\lim_{x\to 0} g(x)$ if $\lim_{x\to -4}\left(x\lim_{x\to 0} g(x)\right)=2$. **02**
2. If gas in a cylinder is maintained at a constant temperature T, the pressure P is related to the volume V by a formula of the form $P=\frac{nRT}{V-nb}-\frac{an^2}{V^2}$, in which a,b,n,R are constants. Find $\frac{dP}{dV}$. **02**
3. For what values of a,m and b does the function **02**
$$f(x)=\begin{cases} 3, & x=0 \\ -x^2+3x+a, & 0<x<1 \\ mx+b & 1\le x\le 2 \end{cases}$$
satisfy the hypotheses of the Mean Value Theorem on the interval [0,2]?
4. Evaluate by L'Hospital rule $\lim_{x\to a}\frac{\log(x-a)}{\log\left(e^x-e^a\right)}$ **02**

(b) If resistors of R_1, R_2 and R_3 ohms are connected in parallel to make an R-ohm resistor, the value of R can be found from the equation $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$. Find the value of $\frac{\partial R}{\partial R_2}$ when $R_1=30, R_2=45$ and $R_3=90$ ohms. **03**

(c) Use the binomial series and the fact that $\frac{d}{dx}\left(\sin^{-1}x\right)=\left(1-x^2\right)^{-\frac{1}{2}}$ to generate the first four nonzero terms of the Taylor series for $\sin^{-1}x$ and hence obtain the first five nonzero terms of the Taylor series for $\cos^{-1}x$. **03**

Q.2

(a)

1. Sketch the region of integration and evaluate it: $\int_1^4\int_0^{\sqrt{x}}\frac{3}{2}e^{\frac{\sqrt{y}}{x}}\,dydx$. **03**
2. The transformation from $r\theta z$ - space to xyz - space is given by the equations $x=r\cos\theta,\ y=r\sin\theta,\ z=z$ find the Jacobian $J\left(r,\theta,z\right)$. **02**
3. Evaluate $\int_C\left(x^2dx+yzdy+\frac{y^2}{2}dz\right)$ along the line segment C joining (0,0,0) to (0,3,4). **02**

(b) 1. You are planning to make an open rectangular box from 8 – in. – by – 15 –in. piece of cardboard by cutting congruent squares from the corners and folding up the sides. What are the dimensions of the box of largest volume you can make this way, and what is its volume? **03**

2. Let $f(x)=1-x^{\frac{2}{3}}$. Show that $f(1)=f(-1)=0$, but that $f'(x)$ is never zero in [-1,1]. Explain how this is possible, in view of Rolle's theorem. **01**

3. For $\sum_{n=0}^{\infty}\frac{(2x+3)^{2n+1}}{n!}$ (a) find the series' radius of convergence. For what values of x does the series converges (b) absolutely, (c) conditionally? **03**

OR

(b) 1. What values of a and b make $f(x)=x^3+ax^2+bx$ have a local minimum at $x=4$ and a point of inflection at $x=1$? **03**

2. Why the function $\begin{cases} \dfrac{\sin x}{x}, & -\pi \le x < 0 \\ 0, & x = 0 \end{cases}$ do not satisfy the hypothesis of Mean Value theorem? Give reason for your answer. **01**

3. For $\sum_{n=0}^{\infty}\frac{(-1)^{n-1}(3x-1)^n}{n^2}$ (a) find the series' radius of convergence. For what values of x does the series converges (b) absolutely, (c) conditionally? **03**

Q.3

(a) Which of the following series converge and which diverge? Give reasons for your answer. **04**

(1) $\sum_{n=1}^{\infty}\frac{1}{n(1+\log^2 n)}$ (2) $\sum_{n=1}^{\infty}\frac{(-1)^n(n+1)^n}{(2n)^n}$

(b) 1. By considering different paths of approach, show that the function $f(x,y)=\frac{x^4-y^2}{x^4+y^2}$ has no limit as $(x,y)\to(0,0)$ **03**

2. Find the gradient of $f(x,y,z)=2z^3-3(x^2+y^2)z+\tan^{-1}(xz)$ at (1,1,1) **02**

3. By using partial derivatives find the value of $\frac{dy}{dx}$ for $xe^y+\sin(xy)+y-\log 2=0$ at $(0,\log 2)$ **02**

(c) Find all the local maxima, local minima, and saddle points of the function $f(x,y)=x^2-2xy+2y^2-2x+2y+1$ **03**

OR

Q.3

(a) Which of the following series converge and which diverge? Give reasons for your answer. **04**

(1) $\sum_{n=1}^{\infty}\frac{n2^n(n+1)!}{3^n n!}$ (2) $\sum_{n=1}^{\infty}\frac{1}{1+2^2+3^2+\ldots+n^2}$

(b) 1. Find the directional derivative of $g(x,y,z)=3e^x\cos(yz)$ at $p_0(0,0,0)$ in the direction of $\vec{A}=2\hat{i}+\hat{j}-2\hat{k}$ **03**

2. Use Taylor's formula for $f(x,y)=\sin x\cos y$ at the origin to find the cubic approximations of f near the origin. **03**

3. Evaluate $\lim\limits_{\substack{(x,y)\to(0,0)\\ x\neq y}} \dfrac{x-y+2\sqrt{x}-2\sqrt{y}}{\sqrt{x}-\sqrt{y}}$ **01**

(c) Find the maximum and minimum values of the function $f(x,y)=3x+4y$ on the circle $x^2+y^2=1$ using the method of Lagrange multipliers. **03**

Q.4

(a)

1. Show that the function $f(x)=\begin{cases}1, & if\ x\ is\ rational\\ 0, & if\ x\ is\ irrational\end{cases}$ has no Riemann integral over [0,1] **02**
2. Find the area of the propeller – shaped region enclosed by the curves $x-y^{\frac{1}{3}}=0$ and $x-y^{\frac{1}{5}}=0$. **03**
3. Discuss Type I and Type II improper integrals with examples of each. Evaluate $\int_2^{\infty}\frac{x+3}{(x-1)(x^2+1)}dx$ **04**

(b) Change the Cartesian integral $\int_0^1\int_0^{\sqrt{1-x^2}} e^{-(x^2+y^2)}dydx$ into an equivalent polar integral. Then evaluate the same. **02**

(c) By using double integration find the area of the region that lies inside the cardioid $r=1+\cos\theta$ and outside the circle $r=1$. **03**

OR

Q.4

(a)

1. A bowl has a shape that can be generated by revolving the graph of $y=x^2/2$ between $y=0$ and $y=5$ about the Y – axis. Find the volume of the bowl. **03**
2. Evaluate the following improper integrals: **04**

   (1) $\int_{-1}^{1}\log|x|\,dx$ (2) $\int_0^{\infty}\frac{dv}{(1+v^2)(1+\tan^{-1}v)}$

(b) Sketch the region of integration, reverse the order of integration and evaluate the integral $\int_0^2\int_0^{4-x^2}\frac{xe^{2y}}{4-y}dydx$ **04**

(c) By using triple integration find the volume of the region between the cylinder $z=y^2$ and the xy- plane that is bounded by the planes $x=0, x=1, y=-1, y=1$. **03**

Q.5

(a) Evaluate $\int_0^4\int_{\frac{y}{2}}^{\frac{y}{2}+1}\frac{2x-y}{2}dxdy$ by applying the transformation $u=\frac{2x-y}{2}, v=\frac{y}{2}$ **04**

and integrating over an appropriate region in the $uv-$plane.

(b) State Green's theorem and use it to find the work done by $\vec{F}=(4x-2y)\hat{i}+(2x-4y)\hat{j}$ in moving a particle once counterclockwise around the circle $(x-2)^2+(y-2)^2=4$. **03**

(c)
1. Find a parametrization of the cylinder $x^2+(y-3)^2=9,\ 0\le z\le 5$ **02**
2. Use the surface integral in Stoke's theorem to calculate the circulation of the field $\vec{F}=\left(y^2+z^2\right)\hat{i}+\left(x^2+z^2\right)\hat{j}+\left(x^2+y^2\right)\hat{k}$ around the curve C: the boundary of the triangle cut from the plane $x+y+z=1$ by the first octant , counterwise when viewed from above. **03**
3. Show that if the necessary partial derivatives of the components of the field $\vec{G}=M\hat{i}+N\hat{j}+P\hat{k}$ are continuous , then $\nabla\cdot\left(\nabla\times\vec{G}\right)=0$. **02**

OR

Q.5

(a) Find a potential function f for the field $\vec{F}=e^{y+2z}\left(\hat{i}+x\hat{j}+2x\hat{k}\right)$ **04**

(b) Obtain Green's theorem Area formula and use it to obtain the area of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. **04**

(c)
1. Use the Divergence theorem to find the outward flux of $\vec{F}=2xz\hat{i}-xy\hat{j}-z^2\hat{k}$ across the boundary of the region D: The wedge cut from the first octant by the plane $y+z=4$ and the elliptical cylinder $4x^2+y^2=16$. **04**
2. Let $f(x,y,z)$ be a function whose second order partial derivatives are continuous then show that $\nabla\times\nabla f=\vec{0}$. **02**

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