Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

B.E. Sem-I Remedial examination March 2009

Subject code: 110008
Subject Name: MATHS - I

Date: 18 / 03 /2009 **Time: 10:30am To 1:30pm**

Instructions: **Total Marks: 70**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1

(a) Do as directed (Each of one mark) **08**

i. State Lagrange's Mean value theorem. What does it geometrically mean?.

ii. Define critical point. Does local extremum exist at $x = 0$ to the function $y = |x|$, however it is not differentiable at $x = 0$?.

iii. For which values of p does the series $\sum_{n=1}^{\infty} \frac{n+1}{n^p}$ is convergent.

iv. Find the radius of convergence for the series $\sum_{n=1}^{\infty} \frac{x^n}{n+2}$

v. Can we solve the integral $\int_0^5 \frac{1}{(x-2)^2} dx$ directly?. Give the reason.

vi. Find the directional derivative of the function $f(x, y) = ax + by; a, b$ are constants, at the point $(0,0)$ which makes an angle of 30° with positive x-axis.

vii. Evaluate the integral $\int_0^{\pi/2} \int_0^{1-\sin\theta} r^2 \cos\theta \, dr \, d\theta$

viii. Find the constants a, b, c so that $\overline{F} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$ is irrotational

(b) Attempt the following

i. If $|x - 1| < \frac{1}{10}$, prove that $|x^3 + 1| < 0.331$ **02**

ii. It can be shown that the inequalities $1 - \frac{x^2}{6} < \frac{x \sin x}{2 - 2\cos x} < 1$ hold for all values of x close to zero. What, if anything, does this tell you about $\lim_{x \to 0} \frac{x \sin x}{2 - 2\cos x}$? **02**

iii. Prove that $f(x) = x - [x], x \in R$ is discontinuous at all integral points. **02**

Q.2

(a) Attempt the following questions

i. Evaluate $\int_{2}^{3}(x-2)dx$ by using an appropriate area formula. **02**

ii. State First Fundamental Theorem of Calculus. Find the value of c by using MVT for integral, for the function $f(x)=\sin x, x\in\left[0,\frac{\pi}{2}\right]$. **02**

iii. Expand $\sin(\frac{\pi}{4}+x)$ in powers of x. Hence find the value of $\sin 44^{o}$. **03**

(b) Attempt the following questions

i. If $x > y > 0$ then prove by LMVT that $\frac{1}{1+x^2}<\frac{\tan^{-1}x-\tan^{-1}y}{x-y}<\frac{1}{1+y^2}$. Hence deduce that $\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\frac{4}{3}<\frac{\pi}{4}+\frac{1}{6}$. **03**

ii. Can the Rolle 's Theorem for $f(x)=|x|, x\in[-1,1]$ applied? **02**

iii. Define stationary point. Suppose that a manufacturing firm produces x number of items. The profit function of the firm is given by $P(x)=-\frac{x^3}{3}+729x-2500.$ Find the number of items that the firm should produce to attain maximum profit. **02**

OR

(b) Attempt the following questions

i. State Rolle 's Theorem. Show that this theorem cannot be applied for $f(x)=[x], x\in[0,2]$ however $f'(x)=0$ for all $x\in(1,2)$. **02**

ii. Verify Cauchy's Mean Value theorem for $\frac{1}{x}$ and $\frac{1}{x^2}, \forall x\in[a,b], a>0$. **02**

iii. What is the necessary condition for the function to have a local extremum?. A soldier placed at a point (3, 4) wants to shoot the fighter plane of an enemy which is flying along the curve $y=x^2+4$ when it is nearest to him. Find such the distance. **03**

Q.3

(a) Attempt the following questions

i. Use LMVT to show that if $x > 0$ and $0<\theta<1$ then $\log_{10}(x+1)=x\frac{\log_{10}e}{1+\theta x}$. **02**

ii. Is $\int_{4}^{\infty}\frac{\sin^2 x}{\sqrt{x}(x-1)}dx$ convergent? **02**

iii. Check the convergence of $\int_{0}^{3}\frac{\cos 3x}{x^{5/2}}dx$. **02**

(b) Test the convergence of the following series

i. $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\ldots\infty.$ **02**

ii. $\sum_{n=1}^{\infty}\frac{n^p}{\sqrt{n+1}+\sqrt{n}}.$ **02**

iii. $\sum_{n=1}^{\infty}\frac{n^3+2}{2^n+2}.$ **02**

iv. $\sum_{n=1}^{\infty}ne^{-n^2}$ **02**

OR

Q.3

(a) Attempt the following questions

i. State Cauchy's Mean Value Theorem. Verify it for $f(x)=\log x, g(x)=\frac{1}{x}, x\in[1,e]$, and find the value of c. **02**

ii. Check the convergence of $\int_4^{\infty}\frac{3x+5}{x^4+7}dx.$ **02**

iii. Find the area between the curve $y^2=\frac{x^2}{1-x^2}$ and its asymptote. **02**

(b) Check the convergence of the following series **08**

i. $\sum_{n=1}^{\infty}\frac{4^n+5^n}{6^n}.$ ii. $\sum_{n=1}^{\infty}\left(\frac{n+1}{n+2}\right)^n x^n$

iii. $1-2x+3x^2-4x^3+\ldots\infty, \quad 0<x<1$

iv. $\sum_{n=1}^{\infty}\frac{(-1)^n}{2n-1}x^{n+1}$

Q.4

(a) Attempt the following questions

i. If $u=\sin^{-1}\left(\frac{x^{1/4}+y^{1/4}}{x^{1/5}+y^{1/5}}\right)$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{20}.$ **02**

ii. If $u=f(x-y, y-z, z-x)$, prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0.$ **03**

iii. Find the extreme values of $x^3+3xy^2-15x^2-15y^2+72x.$ **03**

(b) Attempt the following questions

i. Find the area common to the cardioids $r=a(1-\cos\theta)$ and $r=a(1+\cos\theta)$. **04**

ii. Evaluate $\iint_R (x^2+y^2)dA$, by changing the variables, where R is the region lying in the first quadrant and bounded by the hyperbolas $x^2-y^2=1, x^2-y^2=9, xy=2,$ and $xy=4.$ **02**

OR

Q. 4

(a) Attempt the following questions

i. If $u = \log\left(x^3 + y^3 + z^3 - 3xyz\right)$, prove that **02**

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}.$$

ii. If $u = f(x^2 + 2yz, y^2 + 2zx)$, prove that **03**

$$\left(y^2 - zx\right)\frac{\partial u}{\partial x} + \left(x^2 - yz\right)\frac{\partial u}{\partial y} + \left(z^2 - xy\right)\frac{\partial u}{\partial z} = 0.$$

iii. The temperature at any point *(x, y, z)* in space is $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$ by the method of Lagrange's multipliers. **03**

(b) Attempt the following questions

i. Find the volume generated by the revolution of the loop of the curve $y^2(a + x) = x^2(3a - x)$ about the *x*-axis. **04**

ii. Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} y^2\sqrt{x^2+y^2}\,dydx$, by changing into polar coordinates. **02**

Q.5

(a) Attempt the following questions

i. Evaluate $\iint_R xy\,dA$, where *R* is the region bounded by *x*-axis, ordinate $x = 2a$ and the curve $x^2 = 4ay$. **02**

ii. Evaluate $\iiint_D \sqrt{x^2+y^2}\,dV$, where *D* is the solid bounded by the surfaces $x^2 + y^2 = z^2, z = 0, z = 1$. **03**

(b) Attempt the following questions

i. Find the directional derivative of the divergence of $\overline{F}(x, y, z) = xyi + xy^2 j + z^2 k$ at the point (2,1,2) in the direction of the outer normal to the sphere $x^2 + y^2 + z^2 = 9$. **02**

ii. Prove that $r^n \overline{r}$ is irrotational. **02**

(c) Attempt the following questions

i. Find the work done when a force $\overline{F} = \left(x^2 - y^2 + x\right)i - \left(2xy + y\right)j$ moves a particle in the *xy*-plane from (0, 0) to (1, 1) along the parabola $x^2 = y$?. **02**

ii. Use divergence theorem to evaluate $\iint_S \left(x^3 dydz + x^2 y dzdx + x^2 z dzdx\right)$, where *S* is the closed surface consisting of the cylinder $x^2 + y^2 = a^2$ and the circular discs *z=0* and *z=b*. **03**

OR

Q.5

(a) Attempt the following questions

i. Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dA$ by changing the order of integration. **02**

ii. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx.$ **03**

(b) Attempt the following questions

i. The temperature at any point in space is given by $T = xy + yz + zx$. Determine the derivative of T in the direction of the vector $3i - 4k$ at the point (1, 1, 1). **02**

ii. Show that $\overline{F} = 2xyzi + \left(x^2 z + 2y\right)j + x^2 yk$ is irrotational and find a scalar function ϕ such that $\overline{F} = grad\phi.$ **02**

(c) Attempt the following questions

i. Find $\int_C \overline{F}.\overline{dr}$ where $\overline{F} = \frac{yi - xj}{x^2 + y^2}$ and C is the circle $x^2 + y^2 = 1$ traversed counterclockwise. **02**

ii. Evaluate the surface integral $\iint_S curl\overline{F}.\overline{ds}$ by using Stoke's theorem, where S is the part of the surface of the parabobloid $z = 1 - x^2 - y^2$, for which $z \geq 0$ and $\overline{F} = yi + zj + xk.$ **03**

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