Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

B. E. - SEMESTER –I • EXAMINATION – WINTER 2012

Subject code: 110008 **Date: 21-01-2013**

Subject Name: Mathematics - I

Time: 10.30 am – 01.30 pm **Total Marks: 70**

Instructions:

1. **Attempt any 5 questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) 1 For what value of α **3**

$$f(x) = x^2 + x, \quad x \le 1$$
$$= 2\alpha x, \quad x > 1$$

Continuous at every x ?

2 Determine absolute extrema of **3**

$f(x) = x^2 + x,\ x \in [-5, 5]$.

(b)1 Write all possible functions whose derivative is $3x^2 + 5$. **1**

2 Show that $g(x) = 9x^2 + 2x - 22$ has at least one zero in the interval $\left(-2, -\frac{4}{3}\right)$. **4**

(c) Discuss Maclauarin's series for $f(x) = \sqrt{x}$ and $g(x) = x$. **3**

Q.2 (a)1 State Fundamental Theorem of Calculus and evaluate **2**

$$\frac{d}{dx}\int_0^{x^2} \cos t\, dt$$

2 State the rule which helpful to evaluate $\int_{1/x}^{x} \frac{1}{t} dt$ and then evaluate. **3**

(b)1 Determine volume of a sphere of radius a by the solids of revolution. **3**

2 The region bounded by the curve $y = \sqrt{6x - x^2}$, the x-axis and the line $x = 3$ is revolved about the x-axis to generate a solid. Find the volume of the solid. **3**

(c) Let **3**

$$f(x) = 1 \quad if\ x\ is\ rational\ number$$
$$= \ = \quad if\ x\ is\ irrational\ number$$

Prove or disprove that it is reimann integrable.

Q.3 (a) 1 Determine the convergence of $\int_1^{\infty} \frac{dx}{x^p}$. **4**

2 Prove that $\int_1^{\infty} \frac{5}{e^x + 3} dx$ is convergent. **3**

(b) Expand $f(x) = x^3 - 6x^2 + 14x + 1$ in Taylor's series about $x = 2$. **3**

(c) 1 Evaluate $\lim_{x \to 0} \frac{x - \sin x}{x^3}$. **2**

2 Evaluate $\lim_{x \to \infty} x \tan \frac{1}{x}$ **2**

Q.4 (a) 1 State the Integral Test and determine the convergence of $\sum_{n=1}^{\infty} \frac{2\tan^{-1} n}{1 + n^2}$. **4**

2 Determine the convergence of $\sum_{n=1}^{\infty} \frac{1}{5^n - 1}$. **2**

(b) 1 Investigate the convergence of $\sum_{n=1}^{\infty}\frac{2^n}{n!}$. **2**

2 Show that **3**

$$f(x,y)=\begin{cases}(xy)/(x^2+y^2), & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$$

Is continuous everywhere except at origin.

(c) Solve the system $u=2x+y$, $v=x-2y$ for x and y in terms of u and v. Also find $\frac{\partial(x,y)}{\partial(u,v)}$ **3**

Q.5 (a) 1 Find $\frac{dw}{dt}$ at $t=0$ if $w=xy^2+z^2$ where $x=\cos t,\ y=\sin t,\ z=t$. **3**

2 Let $u=f(x,y)$ is a homogeneous function of degree n. Then find $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$. **4**

(b) Find the directions in which $f(x,y)=\left(x^2/2\right)+\left(y^2/2\right)$ **3**

(a) increases most rapidly at the point (1,1)
(b) decreases most rapidly at (1,1)
(c) what are directions of zero change in f at $(1,1)$?

(c) Define Saddle Point. Find the local extreme values of $f(x)=x^2-y^2-xy-x+y+6$ if possible. **4**

Q. 6 (a)1 Sketch the triangle R in the xy plane bounded by the x axis, the line $y=2x$, and the line $x=1$. **3**

Evaluate $\iint_R \frac{\sin x}{x}\,dA$.

2 Sketch the region of integration and evaluate by reversing the order of integration. **3**

$$\int_0^{3/2}\int_3^{6-2x} x\,dy\,dx$$

(b) Sketch the region of integration and change in to polar integral and then evaluate. **4**

$$\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dy\,dx$$

(c) Find the volume of tetrahedron whose vertices $(0,0,0)$, $(1,0,0)$, $(0,1,0)$, $(0,0,1)$ by triple integral. **4**

Q.7 (a) **4**

1 Evaluate $\int_C (x^2+y)\,ds$ where C is the straight line segment $x=2t$, $y=1-t$, $z=1$ for $0\le t\le 1$.

3

2 Let $\overline{F}=2x^2\hat{i}+xy\,\hat{j}+\hat{k}$ is the velocity field of a fluid in space. Find the flow along the curve $t\hat{i}+t\,\hat{j}+\hat{k}$, $0\le t\le 1$.

(b) **4**

Verify Green's theorem for $\overline{F}=x^2\hat{i}+xy\,\hat{j}$, $C:$ The square bounded by $x=0,\ x=1,\ y=0,\ y=1$.

(c) **3**

Find a parametrization of the cylinder $x^2+(y-2)^2=4,\ 0\le z\le 4$

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