

GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER- 1st / 2nd • EXAMINATION – SUMMER 2013

Subject Code: 110008**Date: 10-06-2013****Subject Name: Maths-I****Time: 02:30 pm – 05:30 pm****Total Marks: 70****Instructions:**

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1 (a) If $3x \leq f(x) \leq x^3 + 2$ for $0 \leq x \leq 2$, Evaluate $\lim_{x \rightarrow 1} f(x)$. 02
 (b) Find the constant c that makes f continuous on $(-\infty, \infty)$, 03

$$f(x) = \begin{cases} x^2 - c^2 & \text{if } x < 4 \\ cx - 20 & \text{if } x \geq 4 \end{cases}$$

- (c) Express the polynomial $x^3 + 7x^2 + x - 6$ in power of $(x - 1)$. 03
 (d) Evaluate: 06

1. $\lim_{x \rightarrow 0} \frac{xe^x - \log(1+x)}{x^2}$.
2. $\lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\tan x}$.

- Q.2 (a) Find the absolute maximum and absolute minimum values of 04
 $f(x) = 3x^2 - 12x + 5$ on the interval $[0, 3]$.

- (b) Expand $e^{\sin x}$ by Maclaurins series up to the terms containing x^4 . 05
 (c) Find two positive numbers whose product is 100 and whose sum is minimum. 05

- Q.3 (a) If $f(x) = x^3 + x^2y^3 - 2y^2$, find $f_x(2, 1)$ and $f_y(2, 1)$. 04

- (b) If $u = \sin^{-1} \frac{x+2y}{x^8+y^8}$, find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$. 05

- (c) Find the local extreme values of $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$. 05

- Q.4 (a) For what values of a , m and b does the function 04

$$f(x) = \begin{cases} 3 & x = 0 \\ -x^2 + 3x + a & 0 < x < 1 \\ mx + b & 1 \leq x \leq 2 \end{cases}$$

Satisfies the hypotheses of the Mean Value theorem on the interval $[0, 2]$?

- (b) Find $\frac{d}{dx} \int_1^{x^2} \cos t \, dt$. 04

- (c) Determine whether series converges or diverges. 06

$$1. \sum_{n=1}^{\infty} \frac{1}{n^2+n+1}. \quad 2. \sum_{n=1}^{\infty} \left(\frac{2n+3}{3n+12} \right)^n.$$

- Q.5 (a) Evaluate: $\int_1^2 \int_y^2 xy \, dx \, dy$. 04

- (b) Evaluate: $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^2 (x^2+y^2) \, dz \, dx \, dy$. 05

- (c) Show that the area between the parabolas $y^2=4ax$ and $x^2=4ay$ is $\frac{16}{3} a^2$. 05

- Q.6 (a) Find the jacobian of the transformation $x = u + 4v$ and $y = 3u - 2v$. 04

- (b) Evaluate the integral $\int_c xy \, dy - y^2 \, dx$, where c is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$. 05

- (c) If $\vec{f} = 3xyi - y^2j$, evaluate $\int \vec{f} \cdot d\vec{r}$, where c is the curve in the xy - plane $y = 2x^2$ from $(0, 0)$ to $(1, 2)$. 05

- Q.7** (a) Find the area enclosed by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. **04**
- (b) Evaluate: **04**
1. $\text{div} [3x^2i + 5xy^2j + xyz^3k]$.
 2. $\text{curl}[e^{xyz}(i + j + k)]$.
- (c) Verify Stoke's theorem for $\vec{f} = (x^2 + y^2)i - 2xyj$ taken around the rectangle **06**
bounded by the lines $x = \pm a$, $y = 0$, $y = b$.
