Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– 1st / 2nd • EXAMINATION – WINTER 2013

Subject Code: 110008 Date: 23-12-2013

Subject Name: Maths-I

Time: 10:30 am – 01:30 pm Total Marks: 70

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) (i) Let the function f be defined as $f(x)=\begin{cases} -1+4x & ; x<-2, \\ -9 & ; x\geq -2 \end{cases}$ **02**

Evaluate $\lim\limits_{x\to -2} f(x)$.

(ii) Verify Rolle's theorem for the function $f(x)=x(x-2)e^{\frac{3x}{4}}$ in [0 2]. **02**

(iii) Verify Lagrange's theorem for the function $f(x)=\cos x$ in [0 π/2]. **02**

(b) (i) Evaluate using L'Hospital's rule $\lim\limits_{x\to 0}\left(\frac{x-\sin x}{x^3}\right)$. **02**

(ii) Evaluate using L'Hospital's rule $\lim\limits_{x\to 0}\left(\frac{a^x-b^x}{x}\right)$. **02**

(iii) Expand $\tan x$ in powers of $\left(x-\frac{\pi}{4}\right)$. **02**

(c) Find the Maclaurin's series for the function $f(x)=e^x$ upto x^4. **02**

Q 2 (a) State the fundamental theorem of integral calculus, part-2 and using it evaluate $\int_0^2 x^2\,dx$. **04**

(b) Trace the curve $r=a(1+\cos\theta)$. **04**

(c) (i) Evaluate $\int_1^\infty \frac{1}{1+x^2}\,dx$. **03**

(ii) Let $f(x)=x+2,\ x\in[0,5]$, find $L_f(P)$ and $U_f(P)$ for $P=\{0,1,2,3,4,5\}$. **03**

Q 3 (a) Test for convergence of the series (i) $\sum_{n=0}^{\infty}\left(\frac{1}{2^n}+\frac{1}{3^n}\right)$ (ii) $\sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$ **04**

(b) State the sandwich theorem for the sequence and using it prove that the sequence $\left\{\frac{\sin n}{n}\right\}_{n=1}^{\infty}$ is convergent and $\lim\limits_{n\to\infty}\left(\frac{\sin n}{n}\right)=0$. **04**

(c) Test for convergence of the series (i) $\sum_{n=1}^{\infty}\frac{n}{e^{-n}}$ (ii) $\sum_{n=1}^{\infty}\left(\frac{1}{1+n}\right)^n$ **06**

Q 4 (a) If $u=\log(\tan x+\tan y+\tan z)$ then prove that **04**

$\sin 2x\frac{\partial u}{\partial x}+\sin 2y\frac{\partial u}{\partial y}+\sin 2z\frac{\partial u}{\partial z}=2$.

(b) Find $J=\frac{\partial(u,v)}{\partial(x,y)}$ where $u=x^2-y^2,\ v=2xy$. **04**

(c) (i) State Euler's theorem for homogeneous functions. If $u = \dfrac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}$ then show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \dfrac{1}{20}u$. **03**

(ii) Discuss about maxima and minima of the function $x^2 + y^2 + 6x - 12$. **03**

Q.5 **(a)** Evaluate $\int_0^2 \int_1^z \int_0^{yz} xyz \, dx \, dy \, dz$. **04**

(b) Evaluate $\iint r \sin\theta \, dr \, d\theta$ over the area of the cardioid $r = a(1+\cos\theta)$ above the initial line. **04**

(c) (i) Find the area of the circle $x^2 + y^2 = 1$ using double integration. **03**

(ii) Evaluate $\int_0^1 \int_0^x e^{\frac{y}{x}} \, dy dx$. **03**

Q.6 **(a)** Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at the point $(2,-1,2)$. **04**

(b) Find the directional derivative of $f(x,y,z) = 4xz^3 - 3x^2y^2z$ at the point $(2,-1,2)$ along the y-axis. **04**

(c) (i) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then prove that $div\,\vec{r} = 0$. **03**

(ii) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then prove that $curl\,\vec{r} = \vec{0}$. **03**

Q.7 **(a)** Verify Green's theorem for $\oint_C \left((y - \sin x)\,dx + \cos x\,dy\right)$ where C is the plane triangle enclosed by the lines $y = 0,\ x = \pi/2,\ y = 2x/\pi$. **04**

(b) Verify Stoke's theorem for $\vec{F} = \left(x^2 + y^2\right)\hat{i} - 2xy\hat{j}$ taken around the rectangle in $xy-$plane bounded by $x = \pm a,\ y = 0,\ y = b$. **04**

(c) Use divergence theorem to evaluate $\iint_S \vec{F} \cdot \hat{n} \, dS$ where $\vec{F} = \left(x^2 - yz\right)\hat{i} + \left(y^2 - xz\right)\hat{j} + \left(z^2 - xy\right)\hat{k}$ and S is the rectangular parallelepiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$ **06**

****