

**GUJARAT TECHNOLOGICAL UNIVERSITY****BE - SEMESTER-1/2 EXAMINATION – WINTER 2017****Subject Code: 110008****Date: 03/01/2018****Subject Name: Maths - I****Time: 10:30 AM TO 01:30 PM****Total Marks: 70****Instructions:**

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** (a) Attempt the following
- (i) Prove that  $f(x) = \begin{cases} x, & x < 0 \\ x^2, & x \geq 0 \end{cases}$  is continuous at  $x = 0$ . 3
- (ii) Find  $c$  of the mean value theorem for  $f(x) = \log_e x; x \in [1, e]$ . 4
- (b) Attempt the following
- (i) Use Taylor's series to find the expansion of  $\log_e x$  in powers of  $(x - 1)$ . 3
- (ii) Evaluate  $\lim_{x \rightarrow 0} \frac{\tan^2 x - x^2}{x^2 \tan^2 x}$ . 4
- Q.2** (a) Attempt the following
- (i) Evaluate  $\lim_{x \rightarrow \pi/2} \cos x \log \tan x$ . 3
- (ii) Find the local extreme values of  $f(x) = x^3 - 9x^2 + 15x + 11$ . 4
- (b) Attempt the following
- (i) Test the convergence of  $\frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \dots \infty$  3
- (ii) Test the convergence of  $\sum_{n=1}^{\infty} \frac{3^n n!}{n^n}$  4
- Q.3** (a) Attempt the following
- (i) Find  $F'(x)$  for  $F(x) = \int_3^{\sin x} \frac{1}{1+t^2} dt$ . 3
- (ii) Check the convergence of  $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$  4
- (b) Find the area of the region between the  $x$ -axis and the graph of  $f(x) = x^3 - x^2 - 2x, -1 \leq x \leq 2$ . 07
- Q.4** (a) If  $u = x^2 y + y^2 z + z^2 x$ , prove that  $(i) \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = 6(x + y + z)$  07
- (ii)  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 2(x + y + z)$
- (b) If  $u = f(2x - 3y, 3y - 4z, 4z - 2x)$ , prove that  $\frac{1}{2} \frac{\partial u}{\partial x} + \frac{1}{3} \frac{\partial u}{\partial y} + \frac{1}{4} \frac{\partial u}{\partial z} = 0$  07

- Q.5 (a)** Attempt the following
- (i) Evaluate  $\iint_R e^{2x+3y} dA$  where R is the triangle bounded by  $x=0$ ,  $y=0$  and  $x+y=1$ . **3**
- (ii) Evaluate  $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dz dy dx$ . **4**
- (b) Evaluate  $\int_0^{2a} \int_{x^2/4a}^{3a-x} (x^2 + y^2) dA$  by changing the order of integration. **07**
- Q.6 (a)** A vector field is given by  $\vec{F} = (x^2 + xy^2)i + (y^2 + x^2y)j$ . Show that  $\vec{F}$  is irrotational and find its scalar potential. **07**
- (b) Verify Gauss' divergence theorem for  $\vec{F} = yi + xj + z^2k$  for the cylindrical region S given by  $x^2 + y^2 = a^2$ ;  $z = 0$  and  $z=h$ . **07**
- Q.7 (a)** Attempt the following
- (i) Using Green's theorem evaluate  $\oint_C [(xy - x^2)dx + x^2ydy]$  along the closed curve C formed by  $y=0$ ,  $x=1$  and  $y=x$ . **3**
- (ii) Find the extreme values for  $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$ . **4**
- (b) Attempt the following
- (i) Expand  $e^x \log(1+y)$  in powers of  $x$  and  $y$ . **3**
- (ii) Find the directional derivative of the function  $\phi = x^2z + 2xy^2 + yz^2$  at the point  $(1,2,-1)$  in the direction of the vector  $\vec{a} = 2i + 3j - 4k$ . **4**

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