Seat No.: ________ Enrolment No.______________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE SEM- I / II Winter Examination-Dec.-2011

Subject code: 110009 **Date: 19/12/2011**

Subject Name: Mathematics-II

Time: 10.30 am -1.30 pm **Total marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1

(a) Find two vectors in R^2 with Euclidean norm 1 whose Euclidean inner product with $(3,-1)$ is zero. **02**

(b) If $v_1 = (1,2,3)$, $v_2 = (-1,-2,-3)$, $v_3 = (2,4,6)$ are three vectors in R^3 that have their initial points at the origin. Do they lie in the same line? **02**

(c) If $(7A)^{-1} = \begin{bmatrix} -3 & 7 \\ 1 & -2 \end{bmatrix}$ then find matrix A. **02**

(d) Is the matrix $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ in row-echelon form or reduced row-echelon form? **02**

(e) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$. **02**

(f) Find the standard matrix for the reflection operator about $xy-$plane in R^3. **02**

(g) Find $u \cdot v$ given that $\|u+v\| = 1$ and $\|u-v\| = 5$ in an inner product space. **02**

Q.2 **(a)**

(I) Show that the set of all pairs of real numbers of the form $(1,x)$ with the operations defined as $(1,x_1)+(1,x_2) = (1,x_1+x_2)$, $k(1,x) = (1,kx)$ is a vector space. **04**

(II) Solve the system of equations $x+y+z=6,\ \ x+2y+3z=14$, $2x+4y+7z=30$ by using Gaussian elimination method. **04**

(b)

(I) Show that the set $S = \{e^x, xe^x, x^2e^x\}$ in $C^2(-\infty,\infty)$ is linearly independent. **03**

(II) Determine whether $W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a,b,c,d \in R,\ a+b+c+d=0 \right\}$ is a subspace of M_{22} or not. **03**

Q.3 (a)

(I) Determine whether the following polynomials span P_2 or not. **03**

$p_1 = 1 - x^2,\ p_2 = 1 + 2x + x^2,\ p_3 = -3x + 2x^2$.

(II) Determine whether the set **03**

$S = \left\{ 2 - x + 4x^2, 3 + 6x + 2x^2, 2 + 10x - 4x^2 \right\}$

is linearly independent or dependent in P_2.

(b)

(I) Find a standard basis vector that can be added to the set **03**

$S = \left\{ (1,-1,0), (3,1,-2) \right\}$ to produce a basis of R^3.

(II) Determine whether b is in the column space of A and if so, express b as a linear combination of the column vectors of A **03**

where $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and $b = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$.

(c) Show that $S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ is a basis of M_{22}. **02**

Q.4 (a)

(I) Determine the dimension and a basis for the solution space of the system $x_1 - 3x_2 + x_3 = 0,\ 2x_1 - 6x_2 + 2x_3 = 0,\ 3x_1 - 9x_2 + 3x_3 = 0$. **03**

(II) Reduce $S = \left\{ (1,-3,2), (2,4,1), (3,1,3), (1,1,1) \right\}$ to obtain a basis for the subspace $W = \text{span } S$ of R^3. **03**

(b)

(I) Find a basis for the row space and column space of the matrix **03**

$$A = \begin{bmatrix} 1 & 4 & 5 & 4 \\ 2 & 9 & 8 & 2 \\ 2 & 9 & 9 & 7 \\ -1 & -4 & -5 & -4 \end{bmatrix}.$$

(II) Find a basis for the null space of $A = \begin{bmatrix} 2 & -1 & -2 \\ -4 & 2 & 4 \\ -8 & 4 & 8 \end{bmatrix}$. **03**

(c) Find rank and nullity of the matrix $\begin{bmatrix} 1 & -1 & -1 \\ 4 & -3 & -1 \\ 3 & -1 & 3 \end{bmatrix}$. **02**

Q.5 (a)

(I) Compute $d(f,g)$ for vectors $f = 2 + 8x^2,\ g = x - 5x^2$ in P_2 with inner product defined as $\langle f, g \rangle = \int_{-1}^{1} f(x) g(x)\, dx$. **03**

(II) Show that the set of vectors $S = \{u_1, u_2, u_3\}$ where **03**

$u_1 = \left(\frac{1}{5}, \frac{1}{5}, \frac{1}{5}\right)$, $u_2 = \left(-\frac{1}{2}, \frac{1}{2}, 0\right)$, $u_3 = \left(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}\right)$ is orthogonal with respect to the Euclidean inner product on R^3. Also convert S into orthonormal set by normalizing the vectors.

(b)

(I) Find a basis for the orthogonal complement of the subspace W of R^3 defined as $W = \{(x, y, z) \in R^3 \mid -2x + 5y - z = 0\}$. **03**

(II) Find a matrix P that diagonalizes the matrix $A = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$ and hence find A^{13}. **03**

(c) If $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ then find the eigen values of A^2 and A^{-1}. **02**

Q.6 (a)

(I) Let R^3 have the Euclidean inner product. Use the Gram-Schmidt process to transform the basis $S = \{(1,1,1), (0,1,1), (0,0,1)\}$ into an orthonormal basis. **03**

(II) Let V be the inner product space of all real valued continuous functions on $\left[0, \pi/2\right]$ with $\langle f, g \rangle = \int_0^{\pi/2} f(x) g(x)\, dx$. Find the angle between $\cos x$ and $\sin x$. **03**

(b)

(I) Determine algebraic and geometric multiplicity of each eigen value of the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$. **03**

(II) Define: (i) Hermitian matrix (ii) Unitary matrix (iii) Normal matrix. **03**

(c) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$ then find the eigen values of A^T and $5A$. **02**

Q.7 (a)

(I) Is $T: R^3 \to R^3$ defined by $T(x, y, z) = (x - y + z, 2y - z, 2x + 3y)$ linear? Is T one to one? **04**

(II) Let $T_1: R^2 \to R^3$, $T_2: R^3 \to R^3$, $T_3: R^3 \to R^2$ be the linear transformations given by $T_1(x, y) = (-2y, 3x, x - 2y)$, $T_2(x, y, z) = (y, z, x)$, $T_3(x, y, z) = (x + z, y - z)$. Find the domain and codomain of $T_3 \circ T_2 \circ T_1$ and find $(T_3 \circ T_2 \circ T_1)(x, y)$. Also find $(T_3 \circ T_2 \circ T_1)(1,1)$. **04**

(b) (I) Express the following quadratic forms in matrix notation. **03**

(i) $2x^2 + 5y^2 - 6z^2 - 2xy - yz + 8zx$

(ii) $2x^2 + 3y^2 + 6xy$

(II) Find the orthogonal projection of $u = (1,2,3,4)$ along $v = (1,-3,4,-2)$ in R^4 with respect to the Euclidean inner product **03**

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