Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

B.E. Sem-I & II Remedial Examination Nov/ Dec. 2010

Subject code: 110009 **Subject Name: Maths - II**

Date: 07/12/2010 **Time: 10:30am- 01:30pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) (i) Verify Cauchy's Schwarz Inequality for the given vectors $u=(-4,2,1)$ and $v=(8,-4,-2)$ **02**

(ii) Use the inner product $\langle p,q\rangle=\int_{-1}^{1}p(x)q(x)dx$ to compute $\langle p,q\rangle$ for vectors $p(x)=1-x+x^2+5x^3$ and $q(x)=x-3x^2$ in P_3. **03**

(iii) Find the ker(T) and determine whether the linear transformation $T:R^2\to R^3$ where $T(x,y)=(x-y,y-x,2x-2y)$ is one to one. **02**

(b) (i) Let R^3 have the Euclidean inner product for which values of k are $u=(k,k,1)$ and $v=(k,5,6)$ are orthogonal? **02**

(ii) Find the standard matrix for the of linear operators on R^2 with rotation of 90^o, followed by reflection about the line y=x. **03**

(iii) Show that the function $T:R^2\to R^2$ given by the formula $T(x_1,x_2)=(x_1+2x_2,3x_1-x_2)$ is a linear operator. **02**

Q.2 (a) (i) Let V be a set of positive real numbers with addition and scalar multiplication defined as $x+y=xy$; $cx=x^c$. Show that V is a vector space under this vector addition and scalar multiplication. **05**

(ii) Express $p(x)=7+8x+9x^2$ as linear combination of $p_1=2+x+4x^2$, $p_2=1-x+3x^2$, $p_3=2+x+5x^2$ **02**

(b) (i) Determine whether the given set of vectors $S=\{1+x,x+x^2,1+x^2\}$ is linearly independent or linearly dependent? **02**

(ii) Determine whether $S=\{a_0+a_1x+a_2x^2+a_3x^3 / a_0=0; a_i\in R, i=0,1,2,3\}$ is a subspace of P_3. **02**

(iii) Determine whether the given vectors $\vec{v}_1=(1,-1,1);, \vec{v}_2=(0,1,2);$ $\vec{v}_1=(3,0,-1)$ forms basis for R^3. **03**

OR

(b) (i) Determine whether set of all functions f such that $f(0)=0$ is subspaces of the space $F(-\infty,\infty)$. **02**

(ii) Verify Pythagorean theorem for the vectors $u=(3,0,1,0,4,-1)$ and $v=(-2,5,0,2,-3,-18)$ **02**

(iii) Determine whether the given vectors $v_1=(2,-1,3);$ $v_2=(4,1,2);$ $v_3=(8,-1,8)$ span R^3. **03**

Q.3 (a) Define rank of a matrix and find rank of given matrix by reducing it to normal form $A=\begin{pmatrix}1&4&5&2\\2&1&3&0\\-1&3&2&2\end{pmatrix}$ **04**

(b) State Dimension Theorem and verify that for the given matrix **04**

$$A=\begin{pmatrix}2 & 2 & -1 & 0 & 1\\ -1 & -1 & 2 & -3 & 1\\ 1 & 1 & -2 & 0 & -1\\ 0 & 0 & 1 & 1 & 1\end{pmatrix}$$

(c) (i) Solve the system of linear equations by Gauss Jordan Elimination Method $-2y+3z=1; 3x+6y-3z=-2; 6x+6y+3z=5$ **03**

(ii) Test for consistency and solve $2x-3y+7z=5; 3x+y-3z=13; 2x+19y-47z=32$ **03**

OR

Q.3 **(a)** (i) Find the basis for null space, Column space and Row space for A **04**

$$A=\begin{pmatrix}2 & -4 & 1 & 2 & -2 & -3\\ -1 & 2 & 0 & 0 & 1 & -1\\ 10 & -4 & -2 & 4 & -2 & 4\end{pmatrix}.$$

(ii) Find the inverse by Gauss Jordan method $A=\begin{pmatrix}2 & 6 & 6\\ 2 & 7 & 6\\ 2 & 7 & 7\end{pmatrix}$ **03**

(b) Find a matrix P that diagonalizes A and determine $P^{-1}AP$, where **04**

$$A=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 1\\ 0 & 1 & 1\end{pmatrix}.$$

(c) Determine whether b is in the column space of A and if so, Express b as linear combination of column vectors of $A=\begin{pmatrix}1 & -1 & 1\\ 9 & 3 & 1\\ 1 & 1 & 1\end{pmatrix}$ and $b=\begin{pmatrix}5\\ 1\\ -1\end{pmatrix}$. **03**

Q.4 **(a)** Let $T: R^2 \to R^2$ be defined by $T\left(\begin{bmatrix}x_1\\ x_2\end{bmatrix}\right)=\begin{pmatrix}x_1+2x_2\\ -x_1\\ 0\end{pmatrix}$ **06**

(i) Find the matrix $[T]_{B'B}$ with respect to the bases B'={u_1, u_2} and B=$\{v_1, v_2, v_3\}$ where $u_1=\begin{pmatrix}1\\ 3\end{pmatrix}$; $u_2=\begin{pmatrix}-2\\ 4\end{pmatrix}$; $v_1=\begin{pmatrix}1\\ 1\\ 1\end{pmatrix}$; $v_2=\begin{pmatrix}2\\ 2\\ 0\end{pmatrix}$;

$v_3=\begin{pmatrix}3\\ 0\\ 0\end{pmatrix}$

(ii) Verify that $[T]_{B'B}\,[x]_B=[T(x)]_{B'}$ holds for every vector $x=\begin{pmatrix}x_1\\ x_2\end{pmatrix}$ in R^2.

(b) Let R^3 have the Euclidean inner product. Use the Gram-Schmidt process to transform the basis $\{u_1, u_2, u_3\}$ into orthonormal basis $u_1=(1,1,1)$, $u_2=(-1,1,0)$ and $u_3=(1,2,1)$. **04**

(c) Let W be the subspace of R^3 spanned by the vectors $w_1=(1,-1,3)$, $w_2=(5,-4,-4)$, $w_3=(7,-6,2)$, Find the orthogonal complement of W. **04**

OR

Q. 4 **(a)** Consider the basis $S=\{v_1,v_2,v_3,v_4\}$ for R^3 where $v_1=(1,1,1)$; $v_2=(1,1,0)$ and $v_3=(1,0,0)$ and Let $T:R^3\to R^3$ be the linear operator such that $T(v_1)=(2,-1,4)$; $T(v_2)=(3,0,1)$; $T(v_3)=(-1,5,1)$. Find a formula for $T(x_1,x_2,x_3)$ and use the formula to find $T(2,4,-1)$ **05**

(b) Find the least square solution of the linear system $Ax=b$ and find the orthogonal projection of b onto the column space of A where $A=\begin{pmatrix}2 & -2\\ 1 & 1\\ 3 & 1\end{pmatrix}$; $b=\begin{pmatrix}2\\ -1\\ 1\end{pmatrix}$ $T:R^2\to R^2$ **05**

(c) Let $T:R^2\to R^2$ is the rotation about the origin through 45° B'=$\{u_1,u_2\}$ and B=$\{v_1,v_2\}$ where $u_1=\begin{pmatrix}1\\ 0\end{pmatrix}$; $u_2=\begin{pmatrix}0\\ 1\end{pmatrix}$; $v_1=\begin{pmatrix}2\\ 1\end{pmatrix}$ and $v_2=\begin{pmatrix}-3\\ 4\end{pmatrix}$ **04**

Q.5 **(a)** Find the characteristic equation of matrix $A=\begin{pmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{pmatrix}$ and verify Caley Hamilton Theorem. **05**

(b) Find a unitary matrix P that diagonalizes A and determine $P^{-1}AP$, where $A=\begin{pmatrix}6 & 2+2i\\ 2-2i & 4\end{pmatrix}$. **05**

(c) (i) Prove that every square matrix A can be expressed as sum of symmetric and skew symmetric matrix **02**

(ii) Is $A=\begin{pmatrix}0 & -3+2i & -2+i\\ 3+2i & 3i & 3+5i\\ 2+i & -3+5i & 2i\end{pmatrix}$ a skew-Hermition matrix? **02**

OR

Q.5 **(a)** Find the eigenvalues and bases for eigenspaces of given matrix **05**

$$A=\begin{pmatrix}0 & 0 & 2 & 0\\ 1 & 0 & 1 & 0\\ 0 & 1 & -2 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$$

(b) Find a matrix P that orthogonally diagonalizes A and determine $P^{-1}AP$ where **05**

$$A=\begin{pmatrix}2 & -1 & -1\\ -1 & 2 & -1\\ -1 & -1 & 2\end{pmatrix}$$

(c) Reduce the quadratic form $Q(x,y,z)=3x^2+3z^2+4xy+8xy+8yz$ into canonical form using linear transformation. **04**

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