

GUJARAT TECHNOLOGICAL UNIVERSITY**B. E. - SEMESTER -I • EXAMINATION - WINTER 2012****Subject code: 110009****Date: 11-01-2013****Subject Name: Mathematics - II****Time: 10.30 am – 01.30 pm****Total Marks: 70****Instructions:**

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1**
- (a) Define dot product of vectors as matrix multiplication. State and prove Cauchy- Schwarz Inequality. Verify it for vectors $u = (1,2,3)$ & $v = (-1,0,3)$ 5
- (b) Define Vector Space over the field K. Check whether the structure $(R^2, +, \cdot)$ is Vector Space over the field R. Where vector addition $\bar{x} + \bar{y} = (x_1 + y_1, x_2 + y_2)$ & Scalar Multiplication $\alpha \cdot \bar{x} = (\alpha^2 x_1, \alpha^2 x_2)$ for vectors $\bar{x} = (x_1, x_2)$ & $\bar{y} = (y_1, y_2)$ 5
- (c) Define Distance between two vectors. Find $d(u \times v, \hat{v})$ for vectors $u = (1, 2, -1)$ & $v = (-2, 1, 2)$, where \hat{v} is unit vector in the direction of vector v. 4
- Q.2**
- (a) Define Linear combination of vectors, Linearly Dependent vectors and Linearly Independent vectors. Check whether vectors $1, \sin^2 x$ & $\cos 2x$ of $F(-\infty, \infty)$ are Linearly Dependent or Linearly Independent vectors. 5
- (b) Define Basis of Vector Space. Find the standard basis vector(s) that can be added to the following set of vectors to produce a basis for R^3 . Set of vectors $\{v_1 = (-1, 2, 3), v_2 = (1, -2, -2)\}$. 5
- (c) Define Sub Space of a vector space. State the necessary and sufficient condition for a subset of a vector space to be subspace. Check whether subset $W = \left\{ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} : \text{where } a, b, c, d \in Z \text{ with } |A| = 0 \right\}$ of a vector space M_{22} is sub space. 4
- Q.3**
- (a) Define the rank of a matrix. Find the rank of the following by reducing to row echelon form. $A = \begin{pmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{pmatrix}$ 5
- (b) Find the basis for row and column spaces of matrix 5
- $$A = \begin{bmatrix} 1 & 4 & 5 & 4 \\ 2 & 9 & 8 & 2 \\ 2 & 9 & 9 & 7 \\ -1 & -4 & -5 & -4 \end{bmatrix}$$
- (c) What is trivial solution of homogeneous system of equations? Solve the homogeneous system of linear equations : 4
- $$2x_1 + 2x_2 - x_3 + x_5 = 0, -x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0,$$
- $$x_1 + x_2 - 2x_3 - x_5 = 0 \text{ \& } x_3 + x_4 + x_5 = 0$$
- Q.4**
- (a) Define Linear Transformation. Consider the basis $S = \{v_1, v_2, v_3\}$ for R^3 . Where $v_1 = (1, 1, 1), v_2 = (1, 1, 0)$ & $v_3 = (1, 0, 0)$. A Linear Transformation $T : R^3 \rightarrow R^2$ such that $T(v_1) = (1, 0), T(v_2) = (2, -1)$ & $T(v_3) = (4, 3)$, then find the formula for 5

Linear Transformation T.

- (b) A Linear Transformation $T : R^2 \rightarrow R^3$ defined by $T(x_1, x_2) = (x_2, -5x_1 + 13x_2, -7x_1 + 16x_2)$. Find the matrix of Transformation T with respect to the bases $B = \{u_1 = (3, 1), u_2 = (5, 2)\}$ for R^2 & $B' = \{v_1 = (1, 0, -1), v_2 = (-1, 2, 2), v_3 = (0, 1, 2)\}$ for R^3 . 5
- (c) Find the standard matrix for the Linear operator on R^3 that 4
 (1) its reflection through the 'xz - plane' (2) rotate each vector 90° counterclockwise about z - axis (along the positive z - axis toward the origin). Check your answer geometrically by sketching the vector $(1, 1, 1)$ and $T(1, 1, 1)$.
- Q.5** (a) Define inner product space. Find the matrix generated the inner product $\langle u, v \rangle = 3u_1v_1 + 5u_2v_2$ 5
- (b) Use Gram- Schmidt Process to transform the basis $\{u_1, u_2, u_3\}$ of R^3 (with usual inner product space) into orthonormal bases. Where $u_1 = (1, 1, 1), u_2 = (0, 1, 1), & u_3 = (0, 0, 1)$. 5
- (c) Find the least square solution of the linear system $AX = B$ given by $x_1 - x_2 = 4, 3x_1 + 2x_2 = 1$ & $-2x_1 + 4x_2 = 3$. 4
- Q. 6** (a) Define Hermitian Matrix and Unitary Matrix. Check whether the given matrix is hermitian matrix or unitary matrix : $A = \begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$. 5
- (b) Find Algebraic Multiplicity & Geometric Multiplicity for the Matrix $A = \begin{bmatrix} 4 & 6 & 6 \\ -8 & -10 & -8 \\ 4 & 4 & 2 \end{bmatrix}$. 5
- (c) Verify Dimension (Rank - Nullity) theorem for a linear Transformation T multiplication by a matrix A given by $A = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 5 & 13 \\ -2 & -1 & -4 \end{bmatrix}$ 4
- Q.7** (a) Find an orthogonal matrix P which diagonalizes the matrix $A = \begin{bmatrix} 1 & 4 \\ 4 & 1 \end{bmatrix}$. 5
- (b) Reduced the quadratic form into canonical form for $2x_1^2 + 5x_2^2 + 5x_3^2 + 4x_1x_2 - 4x_1x_3 - 8x_3x_2$. 5
- (c) What is Consistent solution of non -homogeneous system of equations? Solve the system of linear equations by Gaussian Elimination : $x + y + 2z = 9, 2x + 4y - 3z = 1$ & $3x + 6y - 5z = 0$. 4
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