Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– 1st / 2nd • EXAMINATION – SUMMER 2013

Subject Code: Maths-II **Date: 05-06-2013**

Subject Name: 110009

Time: 02:30 pm – 05:30 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a** Let u =(4,1,2,3) and v = (0,3,8,-2).Evaluate following as directed. **04**

1. Find the norm of u + v.
2. Find the Euclidean inner product of u and v.
3. Find the Euclidean distance between u and v.
4. Find 2u – 3v.

(b) Find the inverse of $A=\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ 1 & 1 & 0\end{bmatrix}$. **05**

(c) If $A = \begin{bmatrix}2 & 2 & 0\\ 2 & 1 & 1\\ -7 & 2 & -3\end{bmatrix}$, Prove that $A^2 - 13A + 12 = \mathbf{0}$. **05**

Q.2 **(a)** Define symmetric and skew symmetric matrix. Express $A = \begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{bmatrix}$ as the sum of the symmetric and the skew symmetric matrix. **07**

(b) Determine whether W= { (a,b,c) / $a^2 + b^2 < 1$ } is a subspace of R^3. **03**

(c) Show that the set of vectors {(2, 1, 1),(1, 2, 2),(1, 1 ,1)} is linearly independent in R^3. **04**

Q.3 **(a)** Find the eigenvalues and eigenvectors of $A = \begin{bmatrix}4 & 0 & 1\\ -2 & 1 & 0\\ -2 & 0 & 1\end{bmatrix}$. **07**

(b) Find the least square solution of the linear system Ax = b given by $x_1 - x_2 = 4, 3x_1 + 2x_2 = 1, -2x_1 + 4x_2 = 3$ and find the orthogonal projection of b on the column space of A. **07**

Q.4 **(a)** Define linear transformation. Find which of the following function are linear transformations. **07**

1. $T : R^3 \to R^3$, Defined by T(x ,y , z) = (x^2 , y , x + y).
2. $T : R^2 \to R^2$, Defined by T(x ,y) = (x ,- y).

(b) Let R^3 have the Euclidean inner product. Use the Gram-Schidt process to transform the basis S = { (1 , 0 ,-1) , (2 , 1, 1) , (0 , -1 , 3) into an orthonormal basis. **07**

Q.5 **(a)** Let V be the set of all positive real number with the operations x + y = x y and k x = x^k , k ∈ R. Show that the set V is a vector space. **07**

(b) Let v_1 = (1 , 2 ,1) , v_2 = (2, 9 , 0) and v_3 = (3 , 3 , 4) **07**

1. Show that the set S = { v_1 , v_2 , v_3 } is a basis for R^3.
2. Find the coordinate vector of v = (5 ,-1 , 9) with respect to S.

Q.6 **(a)** Solve x + y + 2z = 8, – x – 2y + 3z = 1, 3x – 7y + 4z = 10 by Gauss-Jordan elimination. **07**

(b) Find a basis for the null space, the row space and the column space of $A = \begin{bmatrix}1 & -1 & 3\\ 5 & -4 & -4\\ 7 & -6 & 2\end{bmatrix}$. **07**

Q.7 **(a)** Consider the basis S = {v_1,v_2} for R^2,where v_1 = (–2,1) and v_2 = (1,2) and Let T : $R^2 \rightarrow R^3$ be the linear transformation such that T(v_1) = (–1,2,0) and T (v_2) = (0, –3,5).Find a formula for T (x_1,x_2) and use that formula to find T(x_1,x_2). **07**

(b) Define: Real inner product space. Let the vector space P_2 have the inner product space $< p,q > = \int_{-1}^{1} p(x)q(x)\, dx$ then **07**

1. Find $\|p\|$ for $p = 1, q = x$.
2. Find d (p, q) if $p = x, q = x^3$.

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