Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–I & II EXAMINATION – WINTER 2015

Subject Code: 110009 **Date:21/12/2015**

Subject Name: Maths-II

Time: 10:30am to 01:30pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** 1.Let u = (2, -2, 3), v = (1, -3, 4) then **03**

a. Find the norm of u + v.

b. Find the distance between u and v.

c. Find $u \cdot v$

2. Define basis of R^3 and Check the following set S is a basis of R^3 or not. **04**

S= {(1, 2, 1), (2, 1, 0), (-3, 2, 1)}.

(b) Define Vector Space. Let V be the set of all order pair (x, y) of real numbers with operations $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$ and **07**

$K(x_1, y_1) = (K^2 x_1, K^2 y_1)$. Check whether V is a Vector Space over R or not.

Q.2 **(a)** Find the inverse of A (if possible) using row operations, Where **07**

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ -4 & 2 & -9 \end{bmatrix}$$

(b) Determine which of the following are subspace of V. **07**

a. All vectors of the form (x, 0, 0), where $V = R^3$.

b. All matrices of the form $\begin{bmatrix} x & y \\ z & w \end{bmatrix}$,where $x + y + z + w = 0$ and $V=M_{22}$.

Q.3 **(a)** Find bases for the row and column space of **07**

$$A=\begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}$$

(b) Solve the linear system **07**

$-x + y + 2z = 2, 3x - y + z = 6, -x + 3y + 4z = 4.$

Q.4 **(a)** Find the bases for the Eigen space of **07**

$$A=\begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

(b) Determine whether the following functions are Linear Transformation. Justify your answer. **07**

a. Let T: M33 → M_{33}, Defined by T (A) = A^T.

b. Let T: $R^3 \to R^3$, Defined by T (x, y, z) = (x^2, y^2, z^2).

Q.5 **(a)** Consider the basis S= {(-2,1),(1,3)} for R^2 and T:$R^2 \to R^3$ be the Linear Transformation such that T(-2,1) = (-1, 2, 0) and T(1, 3) = (0, -3, 5) **07**

Find a formula for T(x, y) and find T (2, -3).

(b) Define Real inner product Space. Let Vector Space P_2 have the inner product **07**

$$< p, q> = \int_{-1}^{1} p(x)q(x)dx$$

a. Find the norm of p for p =1 and $p = x^2$.

b. Find the distance between $p = 1$and $q = x$.

Q.6 **(a)** Let S={(1, 2, 1),(2, 9, 0),(3, 3, 4)} be the basis for R^3. **07**

a. Find the co-ordinate vector of v= (5, -1, 9) with respect to S.

b. Find the vector v in R^3 whose co-ordinate vector with respect to S is $(v)_s$=(-1, 3, 2).

(b) Define: Symmetric Matrix and Orthogonal Matrix. **07**

Are the following matrices symmetric or orthogonal?

1. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$ 2. $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ 3. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

Q.7 **(a)** Find an orthogonal matrix P that diagonalizes $A=\begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$. **07**

(b) Let R^3 have the Euclidean inner product. Use the Gram-Schmidt process to transform the basis S={(1, 0, 0),(3, 7, -2),(0, 4, 1)} into an orthonormal basis. **07**

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