Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

B. E. - SEMESTER –I • EXAMINATION – WINTER 2012

Subject code: 110014 **Date: 21-01-2013**

Subject Name: Calculus

Time: 10.30 am – 01.30 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a)

(1) Find the Taylor series expansion of $f(x) = \tan x$ in the powers of $\left(x - \frac{\pi}{4}\right)$, showing at least four nonzero terms. Hence find the value of $\tan 46^0$. **[4]**

(2) Find $\lim_{x \to 0} \left(\frac{e^x + e^{2x} + e^{3x}}{3} \right)^{\frac{1}{x}}$ **[3]**

(b)

(1) What is the average value of $f(x) = x^2 - 3x + 2$ over the closed interval $[1,\ 2]$? For what values of x in the given interval, the function will attain it? **[4]**

(2) For $I_n = \int_0^{\pi/2} \cos^n x\ dx$, $n = 2, 3, 4, \ldots$, show that $I_n = \frac{n-1}{n} I_{n-2}$. Also, evaluate $\int_0^{2\pi} \cos^{10} x\ dx$. **[3]**

Q.2 (a)

(1) Show that $\lim_{n \to \infty} \frac{x^n}{n!} = 0$, where $x > 0$ and x remains fixed as n tends to infinity. **[4]**

(2) Test the convergence of $\frac{3}{1^2 - 3} + \frac{3}{2^2 - 3} + \frac{3}{3^2 - 3} + \frac{3}{4^2 - 3} + \ldots$. **[3]**

(b)

(1) Is the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \ldots$ convergent? Does it converge absolutely? Give reasons. **[3]**

(2) Test the convergence of the series $\frac{x}{2 \cdot 5} - \frac{x^2}{2^2 \cdot 10} + \frac{x^3}{2^3 \cdot 15} - \frac{x^4}{2^4 \cdot 20} + \ldots$. Find **[4]**

its radius of convergence.

Q.3 (a)

(1) Determine the intervals over which $f(x)=\frac{x^3}{3}-\frac{x^2}{2}-2x+12$ is monotonically increasing or decreasing. Also find the points of inflection of the function. **[3]**

(2) Discussing main points trace the curve $(a-x)y^2=x^2(a+x)$. **[4]**

(b)

(1) Evaluate the improper integral $\int_5^\infty \frac{5x}{\left(1+x^2\right)^3}dx$ **[3]**

(2) Find the volume of solid of revolution, obtained by rotating the area bounded below the line $2x+3y=6$ in the first quadrant, about the x–axis. **[4]**

Q.4 (a)

(1) Evaluate: $\int_1^4\int_{2x^2}^{3x^2} x\, e^{x^2+y}\, dy\; dx$. **[3]**

(2) Evaluate: $\int_0^1\int_0^y\int_0^{x+2y} (x+y+z)^2 dz\; dx\; dy$ **[4]**

(b)

(1) Evaluate $\iint \left(x^2-y^2\right)^2 dA$, over the area bounded by the lines $|x|+|y|=1$, using the transformations $x+y=u$; $x-y=v$. **[4]**

(2) Evaluate the area between the circles $r=2\cos\theta$ and $r=4\cos\theta$, using double integration. **[3]**

Q.5 (a)

(1) Discuss continuity of $f(x,y)=\begin{cases}\frac{x^2-y^2}{x^2+y^2}; & \text{for}(x,y)\neq(0,0)\\ 0 \quad ; & \text{for}(x,y)=(0,0)\end{cases}$ in $R\times R$ **[3]**

(2) For $u=\tan^{-1}\left(\frac{y}{x}\right)$; show that (a) $\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=0$ (b) $\frac{\partial^2u}{\partial x\partial y}=\frac{\partial^2u}{\partial y\partial x}$ **[4]**

(b)

(1) If $u=f(x-y,\; y-z,\; z-x)$, show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$. **[3]**

(2) For $u = \log\left(x^3 - x^2 y + xy^2 - y^3\right)$, find $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$ **[4]**

Q. 6 (a)

(1) Find the equations of tangent plane and normal line at a point $(3,4,5)$ to the surface $x^2 + y^2 - 4z = 5$. **[3]**

(2) Find the percentage error in T, if l and g are in errors of 2% and 1%, respectively, while measuring T using the formula $T = 2\pi\sqrt{\frac{l}{g}}$. **[4]**

(b)

(1) Find the points of maxima and minima of $f(x,y) = x + y + \frac{1}{x} + \frac{1}{y}$. Also find the maximum and minimum values of the function. **[4]**

(2) Show that $\int_2^{\infty} \frac{5}{e^x - 6}\,dx$ converges. **[3]**

Q. 7 (a)

(1) Using partial differentiation, find $\frac{dy}{dx}$ for $(\cos x)^y = x^{\sin y}$. **[3]**

(2) Find the numbers $x, y, \text{and } z$ such that $xyz = 8$ and $xy + yz + zx$ is maximum, using the Lagrange's method of undetermined multipliers. **[4]**

(b)

(1) Evaluate $\int_0^{\infty}\int_x^{\infty} e^{-y^2}\,dy\,dx$, by changing the order of integration. Also, show the region of integration given by the limits, clearly. **[4]**

(2) Find the volume bounded by the surfaces $z = x^2 + y^2$ and $z = 2$. **[3]**

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