Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– 1st / 2nd • EXAMINATION – SUMMER 2013

Subject Code: 110014 **Date: 10-06-2013**

Subject Name: Calculus

Time: 02:30 pm – 05:30 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) (i) Find the intervals on which the function $f(x) = x^3 - 27x$ is increasing and decreasing. **4**

(ii) Graph the set of points whose polar coordinates satisfy the conditions $1 \le r \le 2$ and $0 \le \theta \le \frac{\pi}{2}$.

(b) Which of following series converge and which diverge? **5**

(i) $\sum_{n=1}^{\infty} \frac{2n+1}{n^2+2n+1}$ (ii) $\sum_{n=1}^{\infty} e^{-n}$

(c) For what values of x do the power series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n-1}}{2n-1}$ converge? **5**

Q.2 (a) (i) Evaluate $\lim_{x \to \frac{\pi}{2}} \frac{2x-\pi}{\cos x}$. **4**

(ii) Find the Maclaurin's series for e^x.

(b) Find the Taylor polynomial of order 3 generated by $f(x) = \sqrt{x}$ at a = 4. **5**

(c) Sketch the curve $y = |x^2 - 1|$ **5**

Q.3 (a) (i) Obtain reduction formula that expresses the integral $\int (\ln x)^n dx$ in terms of an integral of a lower power of $(\ln x)$. **4**

(ii) Find $\frac{dy}{dx}$ if $y = \int_x^5 3t \sin t dt$

(b) State Leibniz's Rule. Use Leibniz's Rule to find the derivative of **5**

$g(y) = \int_{\sqrt{y}}^{2\sqrt{y}} \sin^2 t dt$

(c) Evaluate (i) $\int_0^{\frac{\pi}{4}} \sin^7 2\theta d\theta$ (ii) $\int_0^1 \frac{x^7}{\sqrt{1-x^4}} dx$ **5**

Q.4 (a) (i) Evaluate $\int_0^3 \frac{dx}{x-1}$ if possible. (ii) Evaluate $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ **4**

(b) Find the volume of the solid generated by revolving the region between the parabola $x = y^2$ and the line x = 1 about line x = 1. **5**

(c) Use the shell method to find volume of solid generated by revolving the region bounded by y = x , x = 0 and y =1 about x-axis. **5**

Q.5 (a) (i) Show that $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ does not exist. **4**

(ii) Determine set of all points at which the function

$$f(x,y)=\frac{x^2+y^2}{x-y}$$ is continuous.

(b) Determine whether $u(x,y)=\ln\sqrt{x^2+y^2}$ is a solution of Laplace's equation. **5**

(c) If $f(x,y)=\frac{x^{\frac{1}{4}}+y^{\frac{1}{4}}}{x^{\frac{1}{5}}+y^{\frac{1}{5}}}$ then find $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}$ and **5**

$$x^2\frac{\partial^2 f}{\partial x^2}+2xy\frac{\partial^2 f}{\partial x\partial y}+y^2\frac{\partial^2 f}{\partial y^2}.$$

Q. 6 (a) Find equation for the tangent plane and normal line at point (2,0,2) on the surface $2z - x^2 = 0$ **4**

(b) Find all local maxima, local minima and saddle point of $f(x,y) = x^2 + y^2 + 4x + 6y + 13$ **5**

(c) Suppose that the Celsius temperature at the point (x,y,z) on the sphere $x^2 + y^2 + z^2 = 1$ is $T = 400xyz^2$. Locate the highest and the lowest temperature on the sphere. **5**

Q.7 (a) **4**

Evaluate (i) $\int_1^2\int_0^1 (1+3xy)\,dx\,dy$ (ii) $\int_{-1}^1\int_0^2\int_0^1 xz - y^3\,dz\,dy\,dx$

(b) Find the volume of the solid under the cone $z=\sqrt{x^2+y^2}$ and above the disk $x^2+y^2\le 4$. **5**

(c) **5**

A solid E lies within the cylinder $x^2 + y^2 = 1$, below the plane $z = 4$ and above the paraboloid $z = 1 - x^2 - y^2$. The density at any point is proportional to its distance from the axis of the cylinder. Find mass of E.

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