Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

B. E. - SEMESTER – I • EXAMINATION – WINTER • 2014

Subject code: 110014 **Date: 29-12-2014**

Subject Name: Calculus

Time: 10:30 am - 01:30 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)(i)** Evaluate **03**

(1) $\lim_{x\to 0}\dfrac{2x - x\cos x - \sin x}{x^3}$

(2) $\lim_{x\to 0}\sin x^{\tan x}$

(ii) Using Taylor's series Evaluate $\sin 44^{\circ}$ **03**

(iii) Evaluate $\int_0^a x^{3/2}\sqrt{a-x}\,dx$ **02**

(b)(i) Find the equation of tangent plane and the normal line to the surface $2x^2 + y^2 + 2z = 3$ at (2,1,-3). **03**

(ii) Expand $3x^3 + 8x^2 + x - 2$ in powers of $x-3$. **03**

Q.2 **(a)(i)** Test the convergence of $\int_{-\infty}^{\infty}\dfrac{dx}{1+x^2}$. **03**

(ii) Find the volume of the solid generated by the revolution about the initial line of the cardioids $r = a(1-\cos\theta)$. **04**

(b)(i) Discuss the continuity of **03**

$$f(x,y) = \begin{cases} \dfrac{x^2 - y^2}{x^2 + y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

(ii) If $u = x^3 y + e^{xy^2}$ then prove that $\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{\partial^2 u}{\partial y \partial x}$ **04**

Q.3 **(a)(i)** Find the interval on which the function $f(x)=x^4-4x^3+10$ is increasing and decreasing. **03**

(ii) Trace the curve $r=a(1+\cos\theta),\ a>0.$ **04**

(b)(i) Evaluate $\int_0^{\pi}\sin^2\theta(1+\cos\theta)^4\,d\theta$ **03**

(ii) Find the maximum and minimum value of $y=x^3-9x^2+24x+2$ **04**

Q.4 **(a)(i)** Evaluate $\int_0^1\int_0^{\sqrt{x}}(x^2+y^2)\,dydx$ **03**

(ii) Change the order of integration and evaluate $\int_0^a\int_y^a\frac{x}{x^2+y^2}\,dxdy$ **04**

(b)(i) Evaluate $\int_0^a\int_0^{\sqrt{a^2-x^2}}(x^2+y^2)\,dydx$ by changing into polar coordinates. **03**

(ii) Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}}xyz\,dzdydx$ **04**

Q.5 **(a)(i)** Evaluate (1) $\lim_{n\to\infty}\sqrt[n]{n}$ (2) $\lim_{n\to\infty}\frac{\sin^2 n}{n}$ **03**

(ii) Examine the convergence of **04**

(1) $\sum_{n=1}^{\infty}\frac{2n^2+3n}{5+n^5}$ (2) $\sum_{n=1}^{\infty}\sin\left(\frac{1}{n}\right)$

(b)(i) The region between the curve $y=\sqrt{x}$, $0\le x\le 4$ and the x-axis is revolved about x-axis then find the volume of the solid generated. **03**

(ii) Test the convergence of $x+\frac{x^2}{2^3}+\frac{x^3}{3^3}+\ldots\ldots$ **04**

Q.6 **(a)(i)** If $u=r^m$ *then prove that* **03**

$$\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}+\frac{\partial^2u}{\partial z^2}=m(m+1)r^{m-2}$$

(ii) If $z=f(x,y)$ *and if* $x=e^u+e^{-v},\ y=e^{-u}-e^v$ then prove that **04**

$$\frac{\partial z}{\partial u}-\frac{\partial z}{\partial v}=x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}$$

	(b)(i)	Expand $\tan^{-1}\left(\frac{y}{x}\right)$ in powers of (x-1) and (y-1) using Taylor's	**03**
	(ii)	If $u = x^3 y^2 \sin^{-1}\left(y/x\right)$ *then show that* (1) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 5u$ (2) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 20u$	**04**
Q.7	**(a)(i)**	Find the point on the plane $x + 2y + 3z = 13$ closest to the point (1,1,1).	**03**
	(ii)	if $u = \sin^{-1}\left[\frac{x^{\frac{1}{3}} + y^{\frac{1}{3}}}{x^{\frac{1}{4}} - y^{\frac{1}{4}}}\right]$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{12}\tan u$	**04**
	(b)(i)	Evaluate $\iiint z(x^2 + y^2)dv$ over the volume of the cylinder $x^2 + y^2 = 1$ intercepted by the planes $z = 2$ *and* $z = 3$.	**03**
	(ii)	Find $\frac{dy}{dx}$ if (1) $x^2 + y^3 = 7xy$ (2) $(\cos x)^y = x^{\sin y}$	**04**

****