

**GUJARAT TECHNOLOGICAL UNIVERSITY**  
**BE SEMESTER 1<sup>st</sup> / 2<sup>nd</sup> (OLD) EXAMINATION WINTER 2016**

**Subject Code: 110014****Date: 24/01/2017****Subject Name: Calculus****Time: 10:30 AM TO 1:30 PM****Total Marks: 70****Instructions:**

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1 (a)(1)** Test the convergence of the sequence  $\{2 - (-1)^n a\}$  **03**
- (2)** Show that the sequence  $\{u_n\}$  whose  $n^{\text{th}}$  term is  $u_n = \frac{1}{1!} + \frac{1}{2!} + \dots + \frac{1}{n!}$  where  $n \in N$  **04**  
is monotonic increasing and bounded. Is it convergent?
- (b)** Expand  $\tan^{-1}(x+h)$  in power of  $h$  and hence, find the value of  $\tan^{-1}(1.003)$  up **07**  
to 5 places of decimal.
- Q.2 (a)(1)** Find the expansion of  $\log(1+x)$  **03**
- (2)** Determine whether the following series converge or diverge. Find the sum of **04**  
the series if it converges.  $\sum_{n=1}^{\infty} [\tan^{-1} n - \tan^{-1}(n+1)]$
- (b)** Test the convergence of the series  $\frac{x}{1.2} + \frac{x^2}{3.4} + \frac{x^3}{5.6} + \frac{x^4}{7.8} + \dots$  **07**
- Q.3 (a)(1)** Evaluate :  $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$  **03**
- (2)** Find the linearization of  $f(x, y, z) = x^2 - xy + 3 \sin z$  at the point  $(2, 1, 0)$  **04**
- (b)** Trace the curve  $y^2(a+x) = x^2(b-x)$  **07**
- Q.4 (a)(1)** If  $z = e^{xy}$ ,  $x = t \cos t$ ,  $y = t \sin t$ , find  $\frac{dz}{dt}$  at  $t = \frac{\pi}{2}$  **03**
- (2)** Find the equation of tangent plane and normal line to the surface **04**  
 $x^2 yz + 3y^2 = 2xz^2 - 8z$  at the point  $(1, 2, -1)$ .
- (b)** Change the order of integration and hence evaluate  $\int_0^{4a} \int_{\frac{x^2}{4a}}^{\sqrt{ax}} xy \, dy dx$  **07**
- Q.5 (a)(1)** Find  $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{y^2 - x^2}$  **03**
- (2)** If  $u = \tan^{-1}\left(\frac{x}{y}\right)$  where  $x^2 + y^2 = a^2$  find  $\frac{du}{dx}$ . **04**
- (b)** State Euler's theorem. If  $z = x^n f\left(\frac{y}{x}\right) + y^{-n} f\left(\frac{x}{y}\right)$  prove that **07**  

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} + x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = n^2 z$$

- Q.6 (a)** Find the maxima and minima of the function  $x^3 + y^3 - 63(x + y) + 12xy$  **07**
- (b)** Evaluate  $\iint (x^2 + y^2) dx dy$  over the region bounded by the lines  $y=4x$ ,  $x+y=3$ ,  $y=0$ ,  $y=2$ . **07**
- Q.7 (a)** Evaluate  $\iiint z^2 dx dy dz$  over the region common to the sphere  $x^2 + y^2 + z^2 = 4$  and the cylinder  $x^2 + y^2 = 2x$  **07**
- (b)** Using the method of cylindrical shell, find the volume of the solid obtained by rotating about the x-axis the region under the curve  $y = \sqrt{x}$  from 0 to 1. **07**

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