

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-1/2 EXAMINATION – WINTER 2017****Subject Code: 110014****Date: 03 /01/2018****Subject Name: Calculus****Time: 10:30 AM TO 01:30 PM****Total Marks: 70****Instructions:**

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) (i) Is the following series convergent? Why? 04

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

(ii) Evaluate: 03

$$\int_0^{\pi/2} \sin^2 x \cos^4 x \, dx.$$

(b) (i) Expand $\cos\left(\frac{\pi}{4} + x\right)$ in powers of x . Hence, find the value of $\cos 46^\circ$. 04**(ii) Evaluate 03**

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin x}.$$

Q.2 (a) (i) If $u = \log\left(\frac{x^2+y^2}{x-y}\right)$, then find the value of 04

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}.$$

(ii) If $u = f(xy)$, show that 03

$$x \frac{\partial u}{\partial x} - y \frac{\partial u}{\partial y} = 0.$$

(b) (i) Show that $f(x, y) = \begin{cases} \frac{x^2-y^2}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ is not continuous at $(0, 0)$. 04**(ii) If $x = r \cos \theta$ and $y = r \sin \theta$, find $\frac{\partial x}{\partial r}$ and $\frac{\partial r}{\partial x}$. 03****Q.3 (a) (i) Find the maximum and minimum values and saddle point, if any, of 04**

$$f(x, y) = x^2 + y^2 + 2x + 4y + 9.$$

(ii) Find the equation of tangent plane and normal line at a point $(1, 2, 3)$ to the surface $x^2 + 2y^2 - 3z = 0$. 03**(b) (i) Use Lagrange's method of multipliers to find the maximum value of $f(x, y, z) = xyz$ if it is given that $x + y + z = 24$. 04****(ii) Find the sum of the following series if it converges: 03**

$$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$$

Q.4 (a) (i) Expand $f(x) = \sin x$ by Maclaurin's series. 04**(ii) Test the convergence of the series 03**

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}.$$

- (b) (i) Find the radius of convergence and interval of convergence of the series 04

$$\sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

- (ii) Test the convergence of the series 03

$$\sum_{n=1}^{\infty} \frac{n+1}{n^3+3}.$$

- Q.5** (a) (i) Evaluate $\iint_R xy dA$ over the area between $y = x^2$ and $y = x$. 04

- (ii) Use polar coordinates to evaluate 03

$$\iint_R (x+y) dx dy$$

over the positive quadrant of the circle $x^2 + y^2 = a^2$.

- (b) (i) Change the order of integration and evaluate: 04

$$\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dx dy.$$

03

- (ii) Evaluate:

$$\int_0^1 \int_0^y \int_0^{x+y} y dz dx dy.$$

- Q.6** (a) (i) Trace the curve $r^2 = a^2 \cos 2\theta$. 04

- (ii) Use the fundamental theorem of integral calculus to find $\frac{dy}{dx}$ if 03

$$y = \int_1^{x^2} \cos t dt.$$

- (b) (i) Find the area of the circle $x^2 + y^2 = a^2$ by double integration. 04

- (ii) Find the volume of the region between the curve $y = \sqrt{x}$, $0 \leq x \leq 3$ and the line $x = 3$ revolved about the x -axis to generate a solid. 03

- Q.7** (a) (i) Evaluate the improper integral $\int_0^1 \frac{dx}{\sqrt{x}}$. 04

- (ii) Evaluate: 03

$$\lim_{x \rightarrow \infty} (x)^{\frac{1}{x}}.$$

- (b) (i) Find the area of the region R enclosed by the lines $y = x$, $x = 1$ and the x -axis. 04

- (ii) If $x^2 + y^2 - xy = 0$, find $\frac{dy}{dx}$. 03
