Seat No.: ________ Enrolment No.___________

# GUJARAT TECHNOLOGICAL UNIVERSITY

**BE - SEMESTER–I & II EXAMINATION – WINTER 2015**

**Subject Code: 110015** **Date:21/12/2015**

**Subject Name: Vector Calculus and Linear Algebra**

**Time: 10:30am to 01:30pm** **Total Marks: 70**

**Instructions:**

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

**Q.1** **(a)(i)** Verify the Cauchy-Schwarz inequality **04**

(1) u = (-2,3,1,-1) , v = (7,1,-4,-2)
(2) u = (-3,1,0), v = (2,-1,3)

**(ii)** Given u = (9,3,-4,0,1) , v = (0,-3,2,-1,7) then compute **03**

(1) $u-4v$ (2) u.u (3) $d(u,v)$

**(b)(i)** Use Cramer's rule to solve linear system **04**

$$x+2y+z=5, 3x-y+z=6,\ x+y+4z=7$$

**(ii)** Test the consistency of the system **03**

$$x+2y+z=3, 2x+y+3z=5, 2x+4y+2z=7$$

**Q.2** **(a)(i)** Show that the vectors $v_1=(2,1,1), v_2=(1,2,2), v_3=(1,1,1)$ are Linearly Independent. **04**

**(ii)** Find $A^{-1}$ using row operations if $A=\begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix}$ **03**

**(b)(i)** Express (2,5,-3) as linear combination of $\{(2,-1,1),(1,1,-1),(4,2,1)\}\ in\ R^3$. **04**

**(ii)** Determine whether the following polynomials span $P_2$ **03**

$$p_1(x)=1-x+2x^2, p_2(x)=5-x+4x^2, p_3(x)=-2-2x+2x^2$$

**Q.3** **(a)(i)** Prove that the set of all positive real numbers with operations **05**

$x+y=xy\ and\ k\,x=x^k$ is a real vector space.

**(ii)** For what choices of the parameters the following system have a unique solution ? **02**

$$x+2y=a\ and\ 3x+4y=b$$

**(b)(i)** Find the rank of $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ **04**

**(ii)** Determine if the set of vectors will be a basis for $R^3$. **03**

$v_1 = (1,2,1), v_2 = (2,9,0), v_1 = (3,3,4)$

**Q.4 (a)(i)** A function $T: R^3 \to R^3$ is defined on the basis $B = \{(1,1,1),(1,1,0),(1,0,0)\}\ of\ R^3$ by $T(1,1,1) = (1,2,3), T(1,1,0) = (2,3,4), T(1,0,0) = (3,4,5)\ then\ find\ T(x)\ for\ any\ x.$ **04**

**(ii)** Let $T: R^2 \to R^2$ be defined by $T(x,y) = (x+y, x-y)$ show that T is one-one also find the formula for $T^{-1}(x,y)$. **03**

**(b)(i)** Verify Cayley-Hamilton theorem for the following matrix **04**

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

**(ii)** Determine a basis for the null space of the matrix **03**

$$\begin{bmatrix} 1 & 2 & -3 & 2 & -3 \\ 1 & -1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 1 & -1 \\ 1 & 2 & 5 & -6 & -3 \end{bmatrix}$$

**Q.5 (a)(i)** Evaluate **04**

$$\int_C ydx + xdy + zdz, where\ C\ is\ given\ by\ x = \cos t, y = \sin t, z = t^2, 0 \le t \le 2\pi$$

**(ii)** Evaluate $\int_C F \cdot dr$ where $F = \dfrac{yi - xj}{(x^2+y^2)}$ $and\ C\ is\ the\ circle\ x^2 + y^2 = 1$ traversed counterclockwise. **03**

**(b)(i)** Using Green's theorem evaluate $\oint_C xydy - y^2dx,$ where C is the square cut from the first quadrant by the lines X=1 and Y=1. **04**

**(ii)** Find the value of the constant $\lambda$ such that the vector field defined by **03**

$F = (2x^2y^2 + z^2)i + (3xy^3 - x^2z)j + (\lambda xy^2z + xy)k$ is solenoidal.

**Q.6** **(a)(i)** Find eigen values and eigen vectors of the matrix $A=\begin{bmatrix}1 & 0 & -1\\ 1 & 2 & 1\\ 2 & 2 & 3\end{bmatrix}$. **04**

**(ii)** Determine the algebraic and geometric multiplicity of $A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & -3 & 3\end{bmatrix}$ **03**

**(b)(i)** If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ then using Cayley-Hamilton theorem find $A^2$ *and* $A^{-1}$. **04**

**(ii)** Find the matrix P which diagonalises $A=\begin{bmatrix}2 & 0 & -2\\ 0 & 3 & 0\\ 0 & 0 & 3\end{bmatrix}$. **03**

**Q.7** **(a)(i)** Let $R^4$ have the Euclidian inner product then find the cosine of the angle $\theta$ between vectors $u=(4,3,1,-2), v=(-2,1,2,3)$. **04**

**(ii)** Using Gram-Schmidt process orthonormalize the set of linearly independent vectors $u_1=(1,0,1,1), u_2=(-1,0,-1,1), u_3=(0,-1,1,1)$ *of* $R^4$. with standard inner product. **03**

**(b) (i)** Find the least squares solution of the linear system Ax=b for **04**

$$A=\begin{bmatrix}4 & 0\\ 0 & 2\\ 1 & 1\end{bmatrix}, b=\begin{bmatrix}2\\ 0\\ 11\end{bmatrix}$$

**(ii)** Find the matrix of the quadratic form $x^2+2y^2-7z^2-4xy+8xz+5yz$ . **03**

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