Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE SEMESTER 1st / 2nd (OLD) EXAMINATION WINTER 2016

Subject Code: 110015 **Date:20/01/2017**

Subject Name: Vector Calculus and Linear Algebra

Time:10:30 AM TO 1:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** The shape of a cable from an antenna tower is given by the equation $y = \frac{4}{3}x^{\frac{3}{2}}$ from $x = 0$ to $x = 20$.Find the total length of the cable. **03**

(b) Find the directional derivative of the function $f\,(x, y, z) = x\,y^2 + y\,z^3$ at the point (2,-1,1) in the direction of the vector $i + 2j + 2k$ **04**

(c) Find div $\vec{F}$ and curl $\vec{F}$, where $\vec{F}$ = grad ($x^3 + y^3 + z^3 - 3xyz$). **07**

Q.2 **(a)** Use Green's theorem to evaluate $\int_c (3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy$ where c is the boundary of the region defined by $y = x$ and $y = x^2$. **07**

(b) Use the divergence theorem to evaluate $\iint_s \vec{F}\,.\hat{n}\,ds$ where $\vec{F} = xi + yj + zk$ and s is the portion of the plane $x + 2y + 3z = 6$ which lies in the first octant. **07**

Q.3 **(a)** Find the inverse (if possible) of $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ **03**

(b) Solve the equation $3x + y + 2z = 3$,$2x - 3y - z = -3,\ x + 2y + z = 4$ By Gauss-elimination method. **04**

(c) Reduce the following matrix into reduced raw echelon form and find its rank **07**

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}.$$

Q.4 **(a)** Let u=(-3,2,1,0) , v =(4,7,-3,2) and w = (3,-2,-1,9) **03**

Find(1). 2u + 7v (2). ‖u+w‖ (3). u. v (Euclidean inner product)

(b) Evaluate det (A) where $A = \begin{bmatrix} 21 & 17 & 7 & 10 \\ 24 & 22 & 6 & 10 \\ 6 & 8 & 2 & 3 \\ 6 & 7 & 1 & 2 \end{bmatrix}$ **04**

(c) Check whether V={ (1, x) / $x \in R$ }is a vector space over R with operations (1, y) + (1, z) = (1 , $y + z$) and k (1 , y) = (1 , k y). **07**

Q.5 **(a)** Define (1). Subspace (2). Linear combination (3). Basis **07**

Determine which of the following subspaces of R^3 are.

(1). All vectors of the form (a,0,0) where a∈ R

(2). All vectors of the form (a,1,1) where a∈ R

(c) Find the basis for the row and column space of **07**

$$A = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 \\ 2 & -6 & 9 & -1 & 8 \\ 2 & -6 & 9 & -1 & 9 \\ -1 & 3 & -4 & 2 & -5 \end{bmatrix}$$

Q.6 **(a)** Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ -3 & 5 & 2 \end{bmatrix}$ **07**

(1).Find the eigen value of A.

(2).For each eigen value, find eigen vector corresponding to eigen value.

(3) Is A diagonalizable? Justify your conclusion.

(b) Define: (1) Symmetric matrix (2) Skew symmetric matrix (3) Hermitian matrix (4) Skew- Hermitian matrix (5) Linear transformation **07**

Let T: $R^2 \rightarrow R^3$ is defined by T(x, y) = (x + y, x – y, y), show that T is a Linear transformation

Q.7 **(a)** Define: inner product space. Let R^4 have Euclidean inner product space. Find the cosine of the angle between the vectors u = (4,3,1,-2) and v = (-2,1,2,3). **07**

(b) Let R^3 have Euclidean inner product space. Use the Gram-Schmidt process to transform the basis $\{u_1, u_2, u_3\}$ into an orthonormal basis where $u_1 = (1,1,1)$ $u_2 = (0,1,1)$, $u_3 = (0,0,1)$ **07**

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