Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–1/2 EXAMINATION – WINTER 2017

Subject Code: 110015 **Date:30/12/2017**

Subject Name: Vector Calculus and Linear Algebra

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (1) Find rank of a matrix A= $\begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}$ **03**

(2) Find inverse of A= $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$ using row operations. **04**

(b) (1) Find the determinant of A = $\begin{bmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 2 & 8 & 6 & 1 \end{bmatrix}$ **03**

(2) Solve the system by Gauss Elimination method: **04**

$x + y + 2z = 9,\ 2x + 4y - 3z = 1,\ 3x + 6y - 5z = 0$

Q.2 **(a)** Verify green's theorem for the function F = (x + y) i + 2xy j and C is the rectangle in the xy-plane bounded by x = 0, y = 0, x = a, y = b. **07**

(b) (1) Find directional derivative of the function f (x, y, z) = ax + by; a, b are constants, at the point P (0, 0) which makes an angle of 30^0 with positive x-axis. **03**

(2) Find a potential function for the field $F = e^{y+2z}(i + x\,j + 2x\,k)$. **04**

Q.3 **(a)** Show that the set of all 2 × 2 matrices of the form $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$ with addition defined by $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} + \begin{bmatrix} c & 1 \\ 1 & d \end{bmatrix} = \begin{bmatrix} a+c & 1 \\ 1 & b+d \end{bmatrix}$ and scalar multiplication $k\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} = \begin{bmatrix} ka & 1 \\ 1 & kb \end{bmatrix}$ is a vector space. **07**

(b) (1) Find the basis for the null space of A = $\begin{bmatrix} 2 & 2 & -1 & 0 & 1 \\ -1 & -1 & 2 & -3 & 1 \\ 1 & 1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$ **03**

(2) Show that the set S = $\{v_1, v_2, v_3\}$ is a basis for R^3 where $v_1 = (1, 2, 1)$, $v_2 = (2, 9, 0)$ and $v_3 = (3, 3, 4)$. **04**

Q.4 **(a)** If F: $R^3 \to R^3$ is a linear transformation defined by F (x, y, z, w) = (x + y – z, x – 2y + z, -2x – 2y + 2z). Find basis and dimension of Ker (F) and R(F) **07**

(b) (1) T: $R^3 \to R^3$ is a linear operator defined by the formula T (x, y, z) = (3x+y, -2x-4y+3z, 5x+4y-2z). Show that T is one to one and find $T^{-1}(x, y, z)$ **03**

(2) Consider the basis S = {u, v, w} for R, where u = (1, 1, 1), v = (1, 1, 0) and w = (1, 1, 0). T: $R^3 \to R^2$ is a linear transformation such that T(u) = (1,0), T (v)= (2, -1) and T(w)= (4, 3). Find formula for T (x, y, z) and use it to find T (2, -3, 5) **04**

Q.5 **(a)** Using Gram Schmidt process, construct an orthonormal basis for R basis is the set {(1, 1, 1), (0, 1, 1), (0, 0, 1)} **07**

(b) (1) Find the least square solution of the linear system Ax = b given by x – y = 4, 3x + 2y = 1, -2x +4y = 3 and find the orthogonal projection of b on the column space of A **03**

(2) Let M_{22} have the inner product <A, B> = tr (A^TB). Find the Cosine of the angle between A and B where A = $\begin{bmatrix} 2 & 6 \\ 1 & -3 \end{bmatrix}$, B = $\begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$ **04**

Q.6 **(a)** (1) Find A^{10} for A = $\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$ **03**

(2) Find a matrix p that will diagonalize A = $\begin{bmatrix} 4 & 0 & 1 \\ -1 & -6 & -2 \\ 5 & 0 & 0 \end{bmatrix}$ **04**

(b) Verify Cayley Hamilton theorem for A= $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and use it to find the value of $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$. **07**

Q.7 **(a)** (1) Find the magnitude and the direction of the greatest change of u = xyz^2 at (1, 0, 3) **03**

(2) Find the work done when a force F = $(x^2 - y^2 + x)$ i – (2xy + y) j moves a particle in the xy-plane from (0, 0) and (1, 1) along the parabola $y^2 = x$. Is the work done different when the path is the straight-line y = x? **04**

(b) (1) Find a basis for the subspace of P_2 spanned by the vectors $1 + x$, x^2, $-2 + 2x^2$, $-3x$. **03**

(2) Define: Subspace, symmetric matrix, skew symmetric matrix, orthogonal matrix **04**

****