Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– 1st / 2nd EXAMINATION (New Syllabus) – WINTER 2013

Subject Code: 2110014 **Date: 23-12-2013**

Subject Name: Calculus

Time: 10:30 am – 01:00 pm **Total Marks: 70**

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **Objective Question**

(a) Answer the following questions by choosing the most appropriate answer: **07**

1. What is $\int \frac{x-3}{x} dx$?

(a) $1-3\ln x+C$ (b) $x-3\ln x+C$ (c) $1+\frac{3}{x^2}+C$ (d) $\frac{x^2-3x}{x^2}+C$

2. If $\tan y + x^3 = y^2 + 1$ and $\frac{dx}{dt} = -2$, what is the value of $\frac{dy}{dt}$ at the point (1,0)

(a) -6 (b) -2.5 (c) 0 (d) 6

3. The values of *x* for which the graphs of $y = x$ and $y^2 = 4x$ intersect are

(a) 4 and 4 (b) -4 and 4 (c) 0 and 4 (d) 0 and -4

4. The value of the limit $\lim_{x \to 0} \frac{\tan x}{x}$ is

(a) 0 (b) 1 (c) π (d) ∞

5. If $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ then the derivative of the function *y* w.r.t *x* is

(a) 0 (b) 1 (c) $\frac{2}{(e^x+e^{-x})^2}$ (d) $\frac{4}{(e^x+e^{-x})^2}$

6. If $y = \ln(\sqrt{2}\ x)$ the derivative of the function *y* w.r.t *x* is

(a) $\frac{\sqrt{2}}{x}$ (b) $\frac{1}{\sqrt{2}\,x}$ (c) $\frac{1}{2x}$ (d) $\frac{1}{x}$

7. The sum of the squares of two positive numbers is 200; their minimum product is

(a) 200 (b) $25\sqrt{7}$ (c) 28 (d) none of these

(b) Answer the following questions by choosing the most appropriate answer: **07**

1. The value of the integral $\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx$ is

(a) $-2\cos^{1/2} x + C$ (b) $-\cos\sqrt{x}+C$ (c) $-2\cos\sqrt{x}+C$ (d) $\frac{1}{2}\cos\sqrt{x}+C$

2. The value of the limit $\lim_{x \to 0} \frac{|x|}{x}$ is

(a) 0 (b) 1 (c) π (d) ∞

3. The value of the integral $\int \frac{\ln y}{y^2}$ is

(a) $\frac{1}{y}(1-\ln y)+C$ (b) $\frac{1}{2y}\ln^2 y+C$ (c) $\frac{1}{3y^3}(4\ln y+1)+C$ (d) $-\frac{1}{y}(\ln y+1)+C$

4. For $f(x)=x^2+2x-1,\ 0<x<1$ the value of c in the mean value theorem is

(a) $\frac{1}{2}$ (b) 0 (c) 1 (d) $\frac{1}{3}$

5. The total area bounded by x-axis and $y=\sin x$ is equal to

(a) 4 (b) 1 (c) -1 (d) 0

6. If $A=\begin{bmatrix}-5 & -3\\ 2 & 1\end{bmatrix}$ then A^{-1} is :

(a) $\begin{bmatrix}-5 & -3\\ 2 & -1\end{bmatrix}$ (b) $\begin{bmatrix}-1 & -3\\ 2 & 5\end{bmatrix}$ (c) $\begin{bmatrix}1 & 3\\ -2 & -5\end{bmatrix}$ (d) $\begin{bmatrix}5 & 3\\ -2 & 1\end{bmatrix}$

7. The function $2x^3+3x^2-12x+7$ is decreasing in

(a) [-2, 1] (b) R – [-2,1] (c) [0,2] (d) [1,3]

Q.2 (a) (1) Test the convergence of the series **03**

$$\frac{1.2}{3^2.4^2}+\frac{3.4}{5^2.6^2}+\frac{5.6}{7^2.8^2}+\cdots$$

04

(2) Find value of ***x*** for which the given series $\frac{1}{2\sqrt{1}}+\frac{x^2}{3\sqrt{2}}+\frac{x^4}{4\sqrt{3}}+\frac{x^6}{5\sqrt{4}}+\cdots$ converges.

(b) **03**

(1) Determine convergence or divergence of series $\sum_{n=1}^{\infty}\frac{(2n^2-1)^{\frac{1}{3}}}{(3n^3+2n+5)^{\frac{1}{4}}}$

04

(2) Determine absolute or conditional convergence of the series $\sum_{n=1}^{\infty}(-1)^n.\frac{n^2}{n^3+1}$

Q.3 (a) (1) Find the expansion of $\tan\left(x+\frac{\pi}{4}\right)$ in ascending powers of x upto terms in x^4 and find approximately the value of $\tan 43^0$. **04**

03

(2) Prove that : $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)=\frac{1}{2}\left(x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots\right)$

(b) (1) Evaluate: (i) $\lim_{x\to 0}(\cos x)^{\cot x}$ (ii) $\lim_{x\to\infty}\left(a^{\frac{1}{x}}-1\right)x$ **04**

03

(2) Express $5+4(x-1)^2-3(x-1)^3+(x-1)^4$ in ascending powers of x.

Q.4 (a) (1) Evaluate the iterated integral $\int_0^1\int_x^1 \sin(y^2)\,dy\,dx$ **03**

04

(2) Evaluate the integral $\int_0^{4a}\int_{\frac{y^2}{4a}}^{y}\frac{x^2-y^2}{x^2+y^2}\,dx\,dy$ by transforming into Polar coordinates.

(b) (1) Evaluate the triple integral $\int_0^1\int_0^{\pi}\int_0^{\pi} y\sin z\,dx\,dy\,dz$. **03**

(2) Find the area common to both of the circles $r=\cos\theta$ and $r=\sin\theta$ **04**

Q.5 (a) (1) Determine the set of points at which the given function is continuous **04**

$$f(x,y)=\begin{cases}\dfrac{3x^2y}{x^2+y^2} & if \quad (x,y)\neq(0,0)\\ 0 & if \quad (x,y)=(0,0)\end{cases}$$

(2) If $z=x^2y+3xy^4$, where $x=\sin 2t$ and $y=\cos t$, find $\dfrac{dz}{dt}$ when $t=0$. **03**

(b) (1) If $u=\sin^{-1}\dfrac{x+y}{\sqrt{x}+\sqrt{y}}$ then prove that: **04**

(i) $2x\dfrac{\partial u}{\partial x}+2y\dfrac{\partial u}{\partial y}=\tan u$ (ii) $x^2\dfrac{\partial^2 u}{\partial x^2}+2xy\dfrac{\partial^2 u}{\partial x\partial y}+y^2\dfrac{\partial^2 u}{\partial y^2}=\dfrac{1}{4}\left(\tan^3 u-\tan u\right)$ **03**

(2) If $z=x+y^x$, prove that $\dfrac{\partial^2 z}{\partial x\partial y}=\dfrac{\partial^2 z}{\partial y\partial x}$

Q.6 (a) (1) Find the volume of the solid obtained by rotating the region enclosed by the curves $y=x$ and $y=x^2$ about the x-axis. **03**

(2) Trace the *witch of agnessi* $xy^2=4a^2(a-x)$. **04**

(b) (1) A rectangular box without a lid is to be made from $12m^2$ of cardboard. Find the maximum volume of such a box. **04**

(2) Find the equations of the tangent plane and normal line at the point $(-2,1,-3)$ to the ellipsoid $\dfrac{x^2}{4}+y^2+\dfrac{z^2}{9}=3$. **03**

Q.7 (a) (1) If $u=f\left(\dfrac{x}{y},\dfrac{y}{z},\dfrac{z}{x}\right)$, prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}+z\dfrac{\partial u}{\partial z}=0$. **04**

(2) Evaluate the limit: $\lim\limits_{x\to 1}\left(1-x^2\right)^{\frac{1}{\log(1-x)}}$ **03**

(b) (1) Evaluate $\int_0^a\int_y^a \dfrac{x}{x^2+y^2}\,dx\,dy$ by transforming into polar coordinates. **03**

(2) Find the interval of convergence of the series $\sum\limits_{n=0}^{\infty}\dfrac{(-3)^n x^n}{\sqrt{n+1}}$. **04**

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