GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER- 1st / 2nd • EXAMINATION – SUMMER • 2014

Subject Code: 2110014
Subject Name: Caculus
Time: 02:30 pm - 05:30 pm

Instructions:
1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) 1 The value of \( \lim_{x \to 0} x \sin \frac{1}{x} \) is
(a) 1 (b) \( \pi \) (c) 0 (d) \( \infty \)

2 \( y = \log(\sin x) \) then value of \( \frac{d^2y}{dx^2} = \)
(a) \( \sec^2 x \) (b) \( -\csc^2 x \) (c) \( -\csc x \cot x \) (d) \( \sec x \tan x \)

3 Curve : \( x = t^2 - 1, y = t^2 - t \). For which value of \( t \) tangents of the curve are parallel to the x-axis?
(a) \( t = 0 \) (b) \( t = \frac{1}{\sqrt{3}} \) (c) \( t = \frac{1}{2} \) (d) \( t = -\frac{1}{\sqrt{3}} \)

4 \( \int_{-1}^{1} x |x| \ dx = \) ............
(a) 2 (b) 1 (c) 0 (d) none of these

5 If \( f(x) = 4 - 2x, x < 1 \)
\( = 6x - 4, x \geq 1 \) then find \( \lim_{x \to 1} f(x) \)
(a) \(-2\) (b) 2 (c) 0 (d) doesn’t exist.

6 If \( f: R \to R, f(x) = 3x + 2 \), then find \( f^{-1} \)
(a) not possible (b) \( 3x - 2 \) (c) \( 2x - 3 \) (d) \( \frac{x - 2}{3} \)

7 What is the solution for following equations?
\( 2x + 3y - 5 = 0, 4x + 6y - 7 = 0 \).
(a) no solution (b) infinitely many solution (c) unique solution (d) \{ (0, 0), (-5, -7) \}

(b) 1 \( \lim_{n \to \infty} \frac{1 - n^2}{\sum n} = \)
(a) \(-2\) (b) 1 (c) 2 (d) doesn’t exist.

2 \( \int \frac{1}{\sqrt{x}} \cos \sqrt{x} \ dx = \cdots + c \)
(a) \( \frac{2}{3} \cos^2 x \) (b) \( 2\sqrt{x} \) (c) \( 2\sin \sqrt{x} \) (d) \( \frac{2}{3} \cos \sqrt{x} \)

3 Find the area of the curve \( y = x^2 + 1 \) bounded by x-axis and lines \( x = 1 \) and \( x = 2 \).
(a) \( \frac{3}{10} \) (b) \( \frac{10}{3} \) (c) 6 (d) \( \frac{1}{6} \)

4 \( f(x) = [x], x \in R \) then \( f(x) \) is
(a) Continuous for all real numbers.
(b) Continuous for all integers
(c) discontinuous for all integers
(d) none of above.

5 Find the vertical asymptotes of the curve \( y = \frac{2x^2}{x^2 - 1} \).
(a) \( x = 1 \)  (b) \( x = -1 \)  (c) \( x = \pm 1 \)  (d) \( y = 2 \)

6 \( f(x) = x^3 - 3x^2 + 3x - 100 \) is ……… function.
(a) Increasing (b) decreasing (c) constants (d) none

7 Find the value of \( c \) using Lagrange’s mean value theorem for
\[ f(x) = e^x, x \in [0,1] \]
(a) log1  (b) log e  (c) log(e - 1)  (d) log(e - 0)

Q.2 (a)  (1) Evaluate \( \lim_{x \to 0} \frac{e^x + e^{-x} - x^2 - 2}{\sin^2 x - x^2} \).

(b) (1) Test for convergence the series
\[ \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots \ldots \ldots \]

(b) (2) Expand \( \log \sin x \) in powers of \( (x - 2) \).

Q.3 (a)  (1) Evaluate \( \lim_{x \to 0} (a^x + x)^{\frac{1}{x}} \).
(ii) Test the convergence of \( \sum_{n=0}^{\infty} \frac{2^n}{3^n} \).

(b) (1) Test the convergence of \( \sum_{n=1}^{\infty} \frac{[n+1]^n}{n^{n+1}} \).

Q.4 (a)  (1) If \( u = \ln\left(\frac{z^2 + x^2 + y^2}{x + y + z}\right) \) show that \( \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + x \frac{\partial u}{\partial z} = 6 \).

(b) (1) If \( u = 3xyz \) show that \( \frac{\partial^3 u}{\partial x \partial y \partial z} = (3 + 27xyz + 27x^2 y^2 z^2) e^{3xyz} \).

Q.5 (a)  (1) Find the extreme values of the function
\( f(x, y) = x^3 + y^3 - 3x - 12y + 20 \).

(b) (1) Find the minimum value of \( x^2 y z^3 \) subject to the condition
\[ 2x + y + 3z = \alpha \] using Lagrange’s method of undetermined multipliers.

(2) Expand \( \sin xy \) in powers of \( (x - 1) \) and \( (y - \frac{n}{2}) \) upto second degree terms.
Q.6  (a) (1) Sketch the region of integration, reverse the order of integration and evaluate
\[ \int_{0}^{2} \int_{0}^{\frac{4}{y}} dydx. \]

(2) Evaluate the integral \( \int_{0}^{\frac{\pi}{2}} \int_{0}^{1} \frac{1}{2} \sin \theta \cos \theta drd\theta. \)

(b) (1) Evaluate the integral \( \int_{0}^{2} \int_{0}^{x} \int_{0}^{x} xyz \, dx \, dy \, dz. \)

(2) Evaluate the integral \( \int_{0}^{a} \int_{0}^{y} \int_{0}^{x} y^2 \, dx \, dy \, dz \) by changing into polar co-ordinates.

Q.7  (a) (1) Find the volume bounded by the cylinder \( x^2 + y^2 = 4 \) and the planes \( y + z = 3, z = 0. \)

(2) Evaluate \( \iint_{R} x^2 + y^2 \, dA \) by changing the variables, where \( R \) is the region lying in first quadrant and bounded by the hyperbola \( x^2 - y^2 = 1, x^2 - y^2 = 9, xy = 2 \) and \( xy = 4. \)

(b) 1 The region between the curve \( y = \sqrt{x}, 0 \leq x \leq 4 \) and the x-axis is revolved about the x-axis to generate a solid. Find its volume.

2 Test the convergence of the series \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n \log(n+1)}. \)