Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–1st / 2nd (NEW) EXAMINATION – WINTER 2015

Subject Code: 2110014 **Date:28/12/2015**

Subject Name: Calculus

Time: 10:30am to 01:30pm **Total Marks: 70**

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Sixquestions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

MARKS

Q.1 **Objective Question (MCQ)**

(a) **07**

1. The value of $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ is
   (A) 1 (B) -1 (C) e (D) 1/e
2. The value of $\lim_{x\to 0} x^x$ is
   (A) 1 (B) -1 (C) e (D) 1/e
3. Maclaurin's series of e^{-x} is
   (A) $\sum_{n=0}^{\infty}\frac{x^n}{n!}$ (B) $\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{n!}$ (C) $\sum_{n=1}^{\infty}\frac{x^n}{n!}$ (D) $\sum_{n=1}^{\infty}\frac{(-1)^n x^n}{n!}$
4. If $f(x,y,z,w) = 3\frac{\cos(xw)\sin y^5}{e^y+(1+y^2)/xyw} + 5xzw$, then $\frac{\partial f}{\partial z}$ at (1, 2, 3, 4) is
   (A) 20 (B) 200 (C) 0 (D) 1
5. Minimum value of $f(x,y) = x^2y^2$ is
   (A) 1 (B) 2 (C) 4 (D) 0
6. The sum of the series $\sum_{n=1}^{\infty}\frac{1}{2^n}$ is
   (A) 0 (B) $\frac{3}{4}$ (C) 1 (D) 2
7. Value of the $\lim_{(x,y)\to(0,0)}\frac{x^2-xy}{\sqrt{x}-\sqrt{y}}$ is
   (A) 1 (B) 0 (C) -1 (D) limit does not exist

(b) **07**

1. The value of $\iint 3y\,dxdy$ over the triangle with vertices (-1, 1), (0, 0), and (1, 1) is
   (A) 0 (B) 1 (C) 2 (D) 3
2. The series $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ is
   (A) divergent (B) absolutely convergent (C) conditionally convergent (D) nothing can be said
3. If $J = \frac{\partial(x,\ y)}{\partial(r,\ \theta)}$ and $J^* = \frac{\partial(r,\ \theta)}{\partial(x,\ y)}$, then the value of JJ^* is
   (A) 0 (B) 1 (C) -1 (D) always $J = J^*$
4. What does the polar equation $r = a, a > 0$ represent?
   (A) line (B) rectangle (C) circle (D) parabola
5. The curve $x^3 + y^3 = 3axy$ is symmetric about
   (A) X-axis (B) Y-axis (C) origin (D) the line y = x
6. The value of $\int_1^{\infty}\frac{1}{x^2}\,dx$ is
   (A) 1 (B) 0 (C) -1 (D) does not exist
7. If $f(x,y) = \frac{x}{y} + \sin\left(\frac{y}{x}\right)e^{y/x}$, then the value of $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ is
   (A) 1 (B) 0 (C) -1 (D) f

Q.2 **(a)** Find the equations of tangent plane and normal line to the surface $cos\pi x - x^2y + e^{xz} + yz = 4$ at the point $P(0,1,2)$. **03**

(b) The amount of work done by the heart's main pumping chamber, the left ventricle, is given by the equation **04**

$$W(P,V,\delta,v,g) = PV + \frac{V\delta v^2}{2g}$$

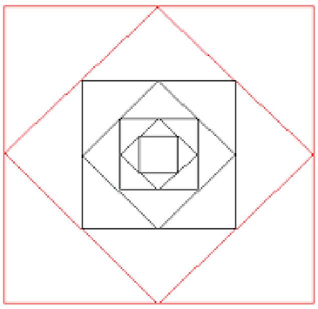
where W is the work per unit time, P is the average blood pressure, V is the volume of blood pumped out during the unit of time, δ is the weight density of the blood, v is the average velocity of the existing blood, and g is the acceleration of the gravity which is constant. Find partial derivative of W with respect to each variable.

(c) Give an example of a non-homogeneous function in x and y. If $u = tan^{-1}(x^2 + xy + 2015y^2)$ then show that **07**

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = sin2u$

(ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2sinu\ cos3u$

Q.3 **(a)** **03**



The figure shows the first seven of a sequence of squares. The outermost square has an area of 4m^2. Each of the other squares is obtained by joining the midpoints of the sides of the squares before it. Find the sum of areas of all squares in the infinite sequence.

(b) Test the convergence of **04**

(i) $\sum_{n=3}^{\infty} \frac{1}{n\, logn\sqrt{log^2 n - 1}}$

(ii) $\sum_{n=1}^{\infty} \frac{1}{1 + 2^2 + 3^2 + \dots + n^2}$

(c) You are to construct an open rectangular box from 12 ft^2 of material. What dimensions will result in a box of maximum volume? **07**

Q.4 **(a)** For $z = tan^{-1}\left(\frac{x}{y}\right)$; $x = ucosv,\ y = usinv$ evaluate $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ at the point $\left(1.3, \frac{\pi}{6}\right)$. **03**

(b) For the series $\sum_{n=1}^{\infty} \frac{(-1)^n (x+2)^n}{n}$ find the series' radius and interval of convergence. For what values of x does the series converge absolutely, conditionally? **04**

(c) Trace the curve $y^2(a - x) = x^3,\ \ a > 0$. **07**

Q.5 **(a)** Evaluate $\lim_{x \to \frac{1}{2}} \frac{cos^2 \pi x}{e^{2x} - 2ex}$ **03**

	(b)	Change the order of integration in $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dydx$ and hence evaluate the same.	04
	(c)	Expand $e^y \log(1+x)$ in powers of x and y up to third degree terms in two different ways.	07
Q.6	(a)	By considering different paths of approach, show that the function $f(x,y) = \frac{x^4 - y^2}{x^4 + y^2}$ has no limit as $(x,y) \to (0,0)$.	03
	(b)	Evaluate $\int_0^3 \frac{dx}{(x-1)^{2/3}}$	04
	(c)	Evaluate $\int_0^4 \int_{\frac{y}{2}}^{\frac{y}{2}+1} \frac{2x-y}{2}\, dxdy$ by applying the transformations $u = \frac{2x-y}{2}, v = \frac{y}{2}$. Draw both the regions.	07
Q.7	(a)	Find the volume of the solid that results when the region enclosed by the curves $y = x^2$ and $x = y^2$ is revolved about the Y-axis.	03
	(b)	Use the method of slicing to find the volume of solid with semicircular base defined by $y = 5\sqrt{cosx}$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. The cross-sections of the solid are squares perpendicular to the X-axis with bases running from the X-axis to the curve.	04
	(c)	Evaluate $\iiint_E 2x dV$ where E is the region under the plane $2x + 3y + z = 6$ that lies in the first octant.	07

****