Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–1/2 (NEW) EXAMINATION – WINTER 2017

Subject Code: 2110014 Date: 03/01/2018

Subject Name: Calculus

Time: 10:30 AM TO 01:30 PM Total Marks: 70

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Mark
Q.1		**Objective Question (MCQ)**	
	(a)	Choose the most appropriate answer out of the following options given for each part of the question:	**07**

1. The value of the $\lim\limits_{x\to\pi} \dfrac{\sin x}{\pi - x}$ is ______

   (A) 0 (B) 1 (C) π (D) -1

2. The sequence $\left\{\left(\dfrac{1}{2}\right)^n\right\}_{n=1}^{\infty}$ converges to ____

   (A) 0 (B) 1 (C) $1/2$ (D) 2

3. The sum of the series $2+\dfrac{2}{3}+\dfrac{2}{9}+\dfrac{2}{27}+\ldots$ is ____

   (A) 3 (B) 0 (C) $1/3$ (D) $2/3$

4. For the curve $y^2(a-x)=x^3, a>0$, the origin is _____

   (A) a Node (B) an isolated point (C) a Cusp (D) None of these

5. If $u=y^2 f\left(x/y\right)$, then $x^2u_{xx}+2xy\,u_{xy}+y^2u_{yy}=$ ____

   (A) 6u (B) 0 (C) u (D) 2u

6. If $J=\dfrac{\partial(u,v)}{\partial(x,y)}$ = 3 then $J^*=\dfrac{\partial(x,y)}{\partial(u,v)}=$ ____

   (A) 3 (B) 1 (C) $1/3$ (D) None of these

7. $\displaystyle\int_0^{\pi/4}\int_0^1 r\ dr d\theta$ = ______

   (A) $\pi/4$ (B) $\pi/8$ (C) $\pi/2$ (D) π

(b) **07**

1. $1-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}-\ldots,\forall x$ is series expansion of the function____

(A) $\sin x$ (B) $\tan x$ (C) e^x (D) $\cos x$

2. The minimum value of $f(x,y)=x^4+y^4+1$ is ____
(A) 3 (B) 0 (C) 1 (D) 16

3. The series $\sum_{n=1}^{\infty}\frac{3n}{5n+1}$ is ________
(A) Convergent (B) Divergent (C) Oscillating (D) can't decide

4. If in the equation of a curve, y occurs only as an even power then the curve is symmetrical about
(A) $x-axis$ (B) $y-axis$ (C) $origin$ (D) None of these

5. A point (a,b) is said to be an extreme point if at (a,b)
(A) $rt-s^2>0$ (B) $rt-s^2<0$ (C) $rt-s^2=0$ (D) $rt-s^2\le 0$

6. The value of $\lim_{x\to\infty} x^{1/x}$ is ______
(A) ∞ (B) 1 (C) 0 (D) None of these

7. The asymptote to the curve $xy^2=4a^2(2a-x), a>0$ is the line _____
(A) $y=0$ (B) $x=2a$ (C) $x=8a^3$ (D) $x=0$

Q.2 **(a)** Expand $3x^3+8x^2+x-2$ in powers of $x-3$. **03**

(b) Evaluate: **04**

(1) $\lim_{x\to 0}(\operatorname{cosec} x-\cot x)$ (2) $\lim_{x\to 1}(2-x)^{\tan\frac{\pi x}{2}}$

(c) Expand $\tan^{-1}(x)$ up to the first four terms by Maclaurin's series and hence prove that $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)=\frac{1}{2}\left(x-\frac{x^3}{3}+\frac{x^5}{5}-\ldots\right)$ **07**

Q.3

(a) Show that the function $f(x,y)=\frac{x^2-y^2}{x^2+y^2}$ has no limit as (x,y) approaches to $(0,0)$ **03**

(b) If $u=f\left(x^2+2yz, y^2+2zx\right)$ then prove that **04**

$$\left(y^2-zx\right)\frac{\partial u}{\partial x}+\left(x^2-yz\right)\frac{\partial u}{\partial y}+\left(z^2-xy\right)\frac{\partial u}{\partial z}=0$$

(c) (1) State Euler's theorem on homogenous function of two variables. **04**

If $u=x^3y^2\sin^{-1}\left(\frac{y}{x}\right)$, show that

(i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=5u$

(ii) $$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=20u$$

03

(2) If resistors of R_1, R_2 and R_3 ohms are connected in parallel to make an R-ohm resistor, such that $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$. Find the value of $\frac{\partial R}{\partial R_2}$ when $R_1=30,\ R_2=45$ and $R_3=90$ ohms.

Q.4 (a) Expand $e^x \cos y$ in powers of x and y up to terms of third degree. **03**

(b) Find the equation of the tangent plane and the normal line of the surface $x^2+2y^2+3z^2-12=0$ at the point $(1,2,-1)$. **04**

(c) Find the shortest and longest distance from the point $(1,2,-1)$ to the sphere $x^2+y^2+z^2=24$ **07**

Q.5

(a) Evaluate $\int_{x=0}^{1}\int_{y=0}^{x}\int_{z=0}^{\sqrt{x+y}} z\,dz\,dy\,dx$ **03**

(b) Evaluate $\iint_R xy\,dA$, where R is the region bounded by $x-axis$, ordinate $x=2a$ and the curve $x^2=4ay$ **04**

(c) Change into polar coordinates and evaluate **07**

$$\int_0^a\int_0^{\sqrt{a^2-y^2}} y^2\sqrt{x^2+y^2}\,dy\,dx$$

Q.6

(a) Discuss the convergence of the integral $\int_1^\infty e^{-x^2}\,dx$. **03**

(b) Check for the convergence of the following: **04**

(1) $\sum_{n=1}^{\infty}\frac{1}{1^2+2^2+\ldots+n^2}$ (2) $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2}$

(c)

(1) Check for convergence of the series $\frac{1}{1.2}-\frac{1}{3.4}+\frac{1}{5.6}-\frac{1}{7.8}+\ldots$ **03**

(2) Find the radius of convergence and interval of convergence of the series **04**

$$1-\frac{1}{2}(x-2)+\frac{1}{2^2}(x-2)^2+\ldots+\left(-\frac{1}{2}\right)^n(x-2)^n+\ldots$$

Q.7 (a) The region between the curve $y=\sqrt{x}\ \ 0\le x\le 4$, and the $x-axis$ is revolved about the $x-axis$ to generate a solid. Find its volume. **03**

04

(b) Expand $\sin\left(\pi/4+x\right)$ in powers of x and hence find the approximate value of $\sin 44^0$.

(c) Trace the curve $xy^2 = a^2(a-x), a > 0$ **07**

****