Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– 1st / 2nd • EXAMINATION – SUMMER • 2014

Subject Code: 2110015 Date: 16-06-2014

Subject Name: Vector Calculus and Linear Algebra

Time: 02:30 pm - 05:30 pm Total Marks: 70

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **Objective Question**

(a) **07**

1. The number of solutions of the system of equations AX = 0 where A is a singular matrix is

(a) 0 (b) 1 (c) 2 (d) infinite

2. Let A be a unitary matrix then A^{-1} is

(a) A (b) $\overline{A}$ (c) A^T (d) $\left(\overline{A}\right)^T$

3. Let $W = span\{\cos^2 x, \sin^2 x, \cos 2x\}$ then the dimension of W is

(a) 0 (b) 1 (c) 2 (d) 3

4. Let P_2 be the vector space of all polynomials with degree less than or equal to two then the dimension of P_2 is

(a) 1 (b) 2 (c) 3 (d) 4

5. The column vectors of an orthogonal matrix are

(a) orthogonal (b) orthonormal (c) dependent (d) none of these

6. Let $T : R^2 \to R^2$ be a linear transformation defined by $T(x, y) = (y, x)$ then it is

(a) one to one (b) onto (c) both (d) neither

7. Let $T : R^3 \to R^3$ be a linear transformation defined by $T(x, y, z) = (y, z, 0)$ then the dimension of $R(T)$ is

(a) 0 (b) 1 (c) 2 (d) 3

(b) **07**

1. If $\|u + v\|^2 = \|u\|^2 + \|v\|^2$ then u and v are

(a) parallel (b) perpendicular (c) dependent (d) none of these

2. $\|u + v\|^2 - \|u - v\|^2$ is

(a) $< u, v >$ (b) $2 < u, v >$ (c) $3 < u, v >$ (d) $4 < u, v >$

3. Let $T : R^3 \to R^3$ be a one to one linear transformation then the dimension of $Ker(T)$ is

(a) 0 (b) 1 (c) 2 (d) 3

4. Let $A = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$ then the eigen values of A^2 are

(a) 1, 2 (b) 1, 4 (c) 1, 6 (d) 1, 16

5. Let $A = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$ then the eigen values of $A + 3I$ are

(a) 1, 2 (b) 2, 5 (c) 3, 6 (d) 4, 7

6. $div\,\bar{r}$ is

(a) 0 (b) 1 (c) 2 (d) 3

7. If the value of line integral $\oint_C \bar{F} \cdot d\bar{r}$ does not depend on path C then $\bar{F}$ is

(a) solenoidal (b) incompressible (c) irrotational (d) none of these

Q.2 **(a)** Solve the following system of equations using Gauss Elimination method **05**

$2x_1 + x_2 + 2x_3 + x_4 = 6$, $6x_1 - x_2 + 6x_3 + 12x_4 = 36$

$4x_1 + 3x_2 + 3x_3 - 3x_4 = 1$, $2x_1 + 2x_2 - x_3 + x_4 = 10$

(b) Find the inverse of $\begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 3 & 2 \\ 2 & 4 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ using Gauss Jordan method **05**

(c) Express $\begin{bmatrix} 4+2i & 7 & 3-i \\ 0 & 3i & -2 \\ 5+3i & -7+i & 9+6i \end{bmatrix}$ as the sum of a hermitian and a skew– hermitian matrix **04**

Q.3 **(a)** Let V be the set of all ordered pairs of real numbers with vector addition defined as $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2 + 1,\ y_1 + y_2 + 1)$. Show that the first five axioms for vector addition are satisfied. Clearly mention the zero vector and additive inverse. **05**

(b) Find a basis for the subspace of P_2 spanned by the vectors $1 + x,\ x^2,\ -2 + 2x^2,\ -3x$ **05**

(c) Express the matrix $\begin{bmatrix} 5 & 1 \\ -1 & 9 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$ **04**

Q.4 **(a)** Consider the basis $S = \{v_1, v_2\}$ for R^2 where $v_1 = (1, 1)$ and $v_2 = (2, 3)$. Let $T: R^2 \to P_2$ be the linear transformation such that $T(v_1) = 2 - 3x + x^2$ and $T(v_2) = 1 - x^2$ then find the formula of $T(a, b)$ **05**

(b) Verify Rank-Nullity theorem for the linear transformation $T: R^4 \to R^3$ defined by $T(x_1, x_2, x_3, x_4) = (4x_1 + x_2 - 2x_3 - 3x_4,\ 2x_1 + x_2 + x_3 - 4x_4,\ 6x_1 - 9x_3 + 9x_4)$ **05**

(c) Find the algebraic and geometric multiplicity of each of the eigen value of $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ **04**

Q.5 **(a)** For $A = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$ and $B = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$ let the inner product on M_{22} be defined as $< A, B > = a_1 a_2 + b_1 b_2 + c_1 c_2 + d_1 d_2$. Let $A = \begin{bmatrix} 2 & 6 \\ 1 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$ then verify Cauchy-Schwarz inequality and find the angle between A and B **05**

(b) Let R^3 have the inner product defined by $< (x_1, x_2, x_3), (y_1, y_2, y_3) > = x_1 y_1 + 2x_2 y_2 + 3x_3 y_3$. Apply the Gram-Schmidt process to transform the vectors $(1, 1, 1)$, $(1, 1, 0)$ and $(1, 0, 0)$ into orthonormal **05**

vectors

(c) Find a basis for the orthogonal complement of the subspace spanned by the vectors $(2, -1, 1, 3, 0)$, $(1, 2, 0, 1, -2)$, $(4, 3, 1, 5, -4)$, $(3, 1, 2, -1, 1)$ and $(2, -1, 2, -2, 3)$ **04**

Q.6 (a) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 6 & -1 & 1 \\ -2 & 5 & -1 \\ 2 & 1 & 7 \end{bmatrix}$ and hence find A^4 **05**

(b) Show that the vector field $\overline{F} = (y \sin z - \sin x)i + (x \sin z + 2yz)j + (xy \cos z + y^2)k$ is conservative and find the corresponding scalar potential **05**

(c) Find the directional derivative of $x^2 y^2 z^2$ at $(1, 1, -1)$ along a direction equally inclined with coordinate axes **04**

Q.7 (a) Verify Green's Theorem for $\oint_C (3x - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the triangle with vertices $(0, 0)$, $(1, 0)$ and $(0, 1)$ **05**

(b) Verify Stoke's Theorem for $\overline{F} = (x + y)i + (y + z)j - xk$ and S is the surface of the plane $2x + y + z = 2$ which is in the first octant **05**

(c) Find the work done when a force $\overline{F} = (x^2 - y^2 + x)i - (2xy + y)j$ moves a particle in the XY-plane from $(0, 0)$ to $(1, 1)$ along the parabola $y^2 = x$ **04**

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