GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER–I & II (NEW) EXAMINATION – WINTER 2015

Subject Code: 2110015
Subject Name: Vector Calculus and Linear Algebra
Date: 21/12/2015

Total Marks: 70

Instructions:
1. Question No. 1 is compulsory. Attempt any four out of remaining Six questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 Objective Question (MCQ)

(a) 07

1. Eigen values of $A^{-1}$ & $A^T$ are same if matrix $A$ is
   (a) Symmetric (b) Orthogonal (c) skew symmetric (d) None of these
2. Rank of $4 \times 4$ invertible matrix is
   (a)1 (b)2 (c)3 (d)4
3. $\mathbf{F}$ is solenoidal vector, If $\text{div}(\mathbf{F})$ is
   (a) $\mathbf{F}$ (b) 1 (c) 0 (d) -1
4. Let $A$ be a hermitian matrix, then $A$ is
   (a) $A^*$ (b) $\bar{A}$ (c) $A^T$ (d) $-A^*$
5. If $A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$ then eigen values of $A^3$ are
   (a) 1,-1 (b) 0,2 (c) 1,1 (d) 0,8
6. Which set from $S_1 = \{a_0 + a_1x + a_2x^2 / a_0 = 0\}$ and $S_2 = \{a_0 + a_1x + a_2x^2 / a_0 \neq 0\}$ is subspace of $P_2$?
   (a) $S_2$ (b) $S_1$ (c) $S_1 \& S_2$ (d) none of these
7. For which value of $k$ vectors $u = (2, 1, 3)$ and $v = (1, 7, k)$ are orthogonal?
   (a) -3 (b) -1 (c) 0 (d) 2

(b) 07

1. Let $T: R^3 \rightarrow R^3$ be one to one linear transformation then the dimension of $\text{ker}(T)$ is
   (a)0 (b) 1 (c) 2 (d)3
2. The column vector of an orthogonal matrix are
   (a) orthogonal (b) orthonormal (c) dependent (d) none of these
3. If $\mathbf{r} = xi+yj+zk$ then $\text{div} \ (\mathbf{r})$ is
   (a) $\mathbf{r}$ (b) 0 (c) 1 (d) 3
4. The number of solution of the system of equation $AX=0$ (where $A$ is a singular matrix) is
   (a) 0 (b) 1 (c) 2 (d) infinite
5. If the value of line integral does not depend on path $C$ then $\mathbf{F}$ is
   (a) solenoidal (b) incompressible (c) irrotational (d) none of these
6. A Cayley-Hamilton theorem hold for _________ matrices only
   (a) singular (b) all square (c) null (d) a few rectangular
7. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ then rank of matrix $A$ is
   (a) 1 (b) 0 (c) 2 (d) 4
Q.2  (a) Determine whether the vector field \( u = y^2 \hat{i} + 2xy \hat{j} - z^2 \hat{k} \) is solenoidal at a point (1,2,1).

(b) Prove that the matrix \( A = \begin{bmatrix} -1 & 2 + i & 5 - 3i \\ 2 - i & 7 & 5i \\ 5 + 3i & -5i & 2 \end{bmatrix} \) is a Hermition and \( iA \) is a skew Hermition matrix.

(c) For which value of \( \lambda \) and \( k \) the following system have (i) no solution (ii) unique solution (iii) an infinite no. of solution.

\[
\begin{align*}
\lambda x + 2y + 3z &= 10 \\
x + 2y + \lambda z &= k \\
x + y + z &= 6
\end{align*}
\]

Q.3  (a) Find the rank of the matrix \[
\begin{bmatrix}
1 & 2 & 3 \\
2 & 3 & 4 \\
3 & 4 & 5
\end{bmatrix}
\]

(b) Find the inverse of matrix \[
\begin{bmatrix}
1 & 2 & 3 \\
2 & 3 & 4 \\
3 & 4 & 5
\end{bmatrix}
\]
by Gauss-Jordan method.

(c) Consider the basis \( S = \{ \mathbf{v}_1, \mathbf{v}_2 \} \) for \( \mathbb{R}^2 \) where \( \mathbf{v}_1 = (-2,1) \), \( \mathbf{v}_2 = (1,3) \) and let \( T: \mathbb{R}^2 \rightarrow \mathbb{R}^3 \) be the linear transformation such that \( T(\mathbf{v}_1) = (-1,2,0) \), \( T(\mathbf{v}_2) = (0,-3,5) \). Find a formula for \( T(x_1,x_2) \) and use the formula to find \( T(2,-3) \).

Q.4  (a) Express \( p(x) = 7 + 8x + 9x^2 \) as linear combination of

\[
p_1 = 2 + x + 4x^2, \quad p_2 = 1 - x + 3x^2, \quad p_3 = 2 + x + 5x^2.
\]

(b) Solve the system by Gaussian elimination method

\[
x + y + z = 6 \quad x + 2y + 3z = 14 \quad 2x + 4y + 7z = 30
\]

(c) Let \( \mathbb{R}^3 \) have standard Euclidean inner product. Transform the basis \( S = \{ \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \} \) into an orthonormal basis using Gram-Schmidt Process where \( \mathbf{v}_1 = (1,1,1), \mathbf{v}_2 = (-1,1,0), \mathbf{v}_3 = (1,2,1) \).

Q.5  (a) Find the nullity of the matrix

\[
\begin{bmatrix}
2 & 0 & -1 \\
4 & 0 & -2 \\
0 & 0 & 0
\end{bmatrix}
\]

(b) Find the least square solution of the linear system \( Ax = b \) and find the orthogonal projection of \( b \) onto the column space of \( A \) where

\[
A = \begin{bmatrix}
2 & -2 \\
1 & 1 \\
3 & 1
\end{bmatrix}, \quad b = \begin{bmatrix}
2 \\
-1 \\
1
\end{bmatrix}
\]
(c) Show that \( S = \begin{bmatrix}
1 & 2 \\
1 & -2 \\
0 & -1 \\
0 & 3 \\
0 & 1 \\
0 & -1 \\
2 \\
\end{bmatrix} \) is a basis for \( M_{22} \).

Q.6  
(a) Verify Pythagorean theorem for the vectors \( u = (3,0,1,0,4,-1) \) and \( v = (-2,5,0,2,-3,-18) \).
(b) Find the unit vector normal to surface \( x^2y + 2xz = 4 \) at the point \( (2,-2,3) \).
(c) Verify Green’s theorem for \( F = x^2\hat{i} + xy\hat{j} \) under the square bounded by \( x=0, x=1, y=0, y=1 \).

Q.7  
(a) Find \( \text{curl} F \) at the point \( (2, 0, 3) \), if \( F = ze^{xy} \hat{i} + 2xy\cos y\hat{j} + (x + 2y)\hat{k} \)
(b) Show that the set \( V = \mathbb{R}^3 \) with the standard vector addition and scalar multiplication defined as \( c(u_1, u_2, u_3) = (0,0,cu_3) \) is not vector space.
(c) Use divergence theorem to evaluate \( \iint_S (x^3 dydz + x^2 ydzdx + x^2 zdxz) \) where \( S \) is the closed surface consisting of the cylinder \( x^2 + y^2 = a^2 \) and the circular discs \( z=0 \) and \( z=b \).

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