Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–1/2 (NEW) EXAMINATION – WINTER 2017

Subject Code: 2110015 **Date: 30/12/2017**

Subject Name: Vector Calculus & Linear Algebra

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**

1. **Make suitable assumptions wherever necessary.**
2. **Figures to the right indicate full marks.**

Q.1 (a) Choose the appropriate answer for the following. **07**

1. Determine whether the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ is in

(a) row echelon form
(b) reduced row echelon form
(c) row echelon form and reduced row echelon form
(d) none of these

2. If $A = \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix}$ then A^{-1} is

(a) $\begin{bmatrix} -5 & 10 \\ -15 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} -1/5 & 2/5 \\ -3/5 & 1/5 \end{bmatrix}$ (c) $\begin{bmatrix} 5 & -10 \\ 15 & -5 \end{bmatrix}$ (d) $\begin{bmatrix} 1/5 & -2/5 \\ 3/5 & -1/5 \end{bmatrix}$

3. The eigen values of a matrix A=$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is

(a)1,1 (b) -1,-1 (c) 1,-1 (d) none of these

4. If $A = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}$ then AB is

(a) $\begin{bmatrix} 1 & 1 \\ 4 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & -1 \\ -4 & -4 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & -3 \\ 6 & 2 \end{bmatrix}$ (d) none of these

5. If A is a matrix of an order 3×7 and rank of A is 3 then nullity of A is

(a) 3 (b) 7 (c) 4 (d) none of these

6. If $\overline{F}$ is irrotational then

(a) $\nabla\overline{F} \neq \overline{0}$ (b) $\nabla \times \overline{F} = \overline{0}$ (c) $\nabla \cdot \overline{F} = 0$ (d) none of these

7. $\overline{i} \times \overline{j}$ is

(a) $\overline{\mathrm{k}}$ (b) $-\overline{\mathrm{k}}$ (c) 0 (d) none of these

(b) Choose the appropriate answer for the following. **07**

1. If $\overline{\mathrm{u}} = (3, -2)$ and $\overline{\mathrm{v}} = (4, 5)$ then $< \overline{\mathrm{u}}, \overline{\mathrm{v}} >$ is

(a) 5 (b) 0 (c) -2 (d) none of these

2. Determine which of the following is linearly dependent

(a) $(1, 2), (3,4)$ (b) $(-1, -2), (-3, -4)$ (c) $(2, 1), (4, 2)$ (d) none of these

3. The system of equations $x + y = 4$ and $2x + 2y = 6$ has

(a) unique solution (b) no solution (c) infinitely many solution (d) none of these

4. If $A = \begin{bmatrix} 2 & 5 \\ 0 & -2 \end{bmatrix}$ then eigen values of A^2 is

(a) 4, 4 (b) 4, -4 (c) 2, 2 (d) none of these

5. If $\overline{F} = x\overline{i} + y\overline{j} + z\overline{k}$ then $\nabla \cdot \overline{F}$ at (1,1,1) is

(a) 0 (b) -1 (c) 3 (d) none of these

6. If $\overline{u} = 6\overline{i} - 3\overline{j} + 2\overline{k}$ then $\|\overline{u}\|$ is

(a) $\sqrt{49}$ (b) $-\sqrt{49}$ (c) 49 (d) none of these

7. If $\overline{a} \cdot \overline{b} = 0$ then angle between $\overline{a}$ and $\overline{b}$ is

(a) 0 (b) 2π (c) π (d) none of these

Q.2

(a) Find A^{-1} for $A = \begin{bmatrix} 0 & 1 & -1 \\ 3 & 1 & 1 \\ 1 & 2 & -1 \end{bmatrix}$, if exist. **03**

(b) Determine whether the vector $\overline{v} = (-5, 11, -7)$ is a linear combination of the vectors $\overline{v_1} = (1, -2, 2)$, $\overline{v_2} = (0, 5, 5)$ and $\overline{v_3} = (2, 0, 8)$. **04**

(c) When subjected to heat aluminium reacts with copper oxide to produce copper metal and aluminium oxide according to the equation **07**
$Al_3 + CuO \rightarrow Al_2O_3 + Cu$
Using Gauss elimination method, balance the chemical equation.

Q.3

(a) Find the rank of a matrix $A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$. **03**

(b) Solve the system of equation by Gauss-Jordan elimination method, if exist. **04**
$x + 4y - 3z = 0;\ -x - 3y + 5z = -3;\ 2x + 8y - 5z = 1.$

(c) Let $V = \{(a, b): a, b \in \mathbb{R}\}$. Let $\overline{u} = (u_1, u_2)$ and $\overline{v} = (v_1, v_2)$. Define $(u_1, u_2) + (v_1, v_2) = (u_1 + v_1 + 1, u_2 + v_2 + 1)$ and $c(u_1, u_2) = (cu_1 + c - 1, cu_2 + c - 1)$. Verify that V is a vector space. **07**

Q.4

(a) Is $T: \mathbb{R}^2 \rightarrow \mathbb{R}$, defined by $T(x, y) = x^2 + y^2$ linear? **03**

(b) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}$ be the mapping defined by $T(\overline{v}) = Av$ with $A = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$. Show that T is one-to-one. **04**

(c) Fine the eigen values and eigen vectors of $A = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix}$. **07**

Q.5

(a) If $\overline{r} = x\overline{i} + y\overline{j} + z\overline{k}$, show that $\nabla log r = \frac{1}{r}\hat{r}$. Where $\hat{r}$ is unit vector. **03**

(b) Find the directional derivative of $\emptyset = 4xz^3 - 3x^2y^2z$ at the point $(2, -1, 2)$ in the direction $2\overline{i} + 3\overline{j} + 6\overline{k}$. **04**

(c) Let B be the basis for $\mathbb{R}^3$, given by $B = \{(1, 1, 1), (-1, 1, 0), (-1, 0, 1)\}$. Apply the Gram-Schmidt process to B to find an orthonormal basis for $\mathbb{R}^3$. **07**

Q.6

(a) Find constant a, b, c, so that $\overline{F} = (x + 2y + az)\overline{i} + (bx - 3y - z)\overline{j} + (4x + cy + 2z)\overline{k}$ is irrotational. **03**

(b) Determine whether $V = \mathbb{R}^2$ is an inner product space under the inner product $< \overline{u}, \overline{v} > = u_1^2v_1^2 + u_2^2v_2^2$. **04**

(c) Verify the Green's theorem in the plane for $\oint_C (y^2dx + x^2dy)$, where C is the triangle bounded by $x = 0; x + y = 1$; and $y = 0$. **07**

Q.7

(a) Determine whether $\overline{v_1} = (2, 2, 2)$; $\overline{v_2} = (0, 0, 3)$ and $\overline{v_3} = (0, 1, 1)$ span the vector space $\mathbb{R}^3$. **03**

(b) Consider the basis $S = \{\overline{v_1}, \overline{v_2}\}$ for $\mathbb{R}^2$, where $\overline{v_1} = (-2, 1)$ and $\overline{v_2} = (1, 3)$ and let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be the linear transformation such that $T(\overline{v_1}) = (-1, 2, 0)$ and $T(\overline{v_2}) = (0, -3, 5)$. Find a formula for $T(x_1, x_2)$. **04**

(c) Verify rank nullity theorem for $\begin{bmatrix} 1 & 4 & 5 & 0 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix}$ **07**

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