GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER–1/2 (NEW) EXAMINATION – WINTER 2017

Subject Code: 2110015 Date: 30/12/2017
Subject Name: Vector Calculus & Linear Algebra Time: 10:30 AM TO 01:30 PM Total Marks: 70

Instructions:
1. Question No. 1 is compulsory. Attempt any four out of remaining Six questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) Choose the appropriate answer for the following.

1. Determine whether the matrix \[
\begin{bmatrix}
1 & 2 & 2 \\
0 & 0 & 1
\end{bmatrix}
\] is in
   (a) row echelon form
   (b) reduced row echelon form
   (c) row echelon form and reduced row echelon form
   (d) none of these 07

2. If \[
A = \begin{bmatrix}
1 & -2 \\
3 & -1
\end{bmatrix}
\] then \(A^{-1}\) is
   (a) \[
\begin{bmatrix}
-5 & 10 \\
-15 & 5
\end{bmatrix}
\]
   (b) \[
\begin{bmatrix}
-1/5 & 2/5 \\
-3/5 & 1/5
\end{bmatrix}
\]
   (c) \[
\begin{bmatrix}
5 & -10 \\
15 & -5
\end{bmatrix}
\]
   (d) \[
\begin{bmatrix}
1/5 & -2/5 \\
3/5 & -1/5
\end{bmatrix}
\]

3. The eigen values of a matrix \[
A = \begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix}
\] is
   (a) 1, 1
   (b) -1, 1
   (c) 1, -1
   (d) none of these

4. If \[
A = \begin{bmatrix}
1 & -1 \\
2 & 2
\end{bmatrix}
\]
   and \(B = \begin{bmatrix}
1 & -1 \\
2 & 2
\end{bmatrix}\) then \(AB\) is
   (a) \[
\begin{bmatrix}
1 & 1 \\
4 & 4
\end{bmatrix}
\]
   (b) \[
\begin{bmatrix}
-1 & -1 \\
-4 & -4
\end{bmatrix}
\]
   (c) \[
\begin{bmatrix}
-1 & -3 \\
6 & 2
\end{bmatrix}
\]
   (d) none of these

5. If \(A\) is a matrix of an order 3 \(
\times\) 7 and rank of \(A\) is 3 then nullity of \(A\) is
   (a) 3
   (b) 7
   (c) 4
   (d) none of these

6. If \(F\) is irrotational then
   (a) \(\nabla \cdot F \neq 0\)
   (b) \(\nabla \times F = 0\)
   (c) \(\nabla \cdot F = 0\)
   (d) none of these

7. \(\hat{i} \times \hat{j}\) is
   (a) \(\hat{k}\)
   (b) \(-\hat{k}\)
   (c) 0
   (d) none of these

(b) Choose the appropriate answer for the following.

1. If \(\vec{u} = (3, -2)\) and \(\vec{v} = (4, 5)\) then \(<\vec{u}, \vec{v}>\) is
   (a) 5
   (b) 0
   (c) -2
   (d) none of these

2. Determine which of the following is linearly dependent
   (a) (1, 2), (3, 4)
   (b) (-1, -2), (-3, -4)
   (c) (2, 1), (4, 2)
   (d) none of these

3. The system of equations \(x + y = 4\) and \(2x + 2y = 6\) has
   (a) unique solution
   (b) no solution
   (c) infinitely many solution
   (d) none of these

4. If \(A = \begin{bmatrix}
2 & 5 \\
0 & -2
\end{bmatrix}\) then eigen values of \(A^2\) is
   (a) 4, 4
   (b) 4, -4
   (c) 2, 2
   (d) none of these

5. If \(\vec{F} = xi + yj + zk\) then \(\nabla \cdot \vec{F}\) at (1,1,1) is
   (a) 0
   (b) -1
   (c) 3
   (d) none of these

6. If \(\vec{u} = 6\hat{i} - 3\hat{j} + 2\hat{k}\) then \(||\vec{u}||\) is
   (a) \(\sqrt{49}\)
   (b) \(-\sqrt{49}\)
   (c) 49
   (d) none of these

7. If \(\vec{a} \cdot \vec{b} = 0\) then angle between \(\vec{a}\) and \(\vec{b}\) is
   (a) 0
   (b) \(2\pi\)
   (c) \(\pi\)
   (d) none of these
Q.2
(a) Find $A^{-1}$ for $A = \begin{bmatrix} 0 & 1 & -1 \\ 3 & 1 & 1 \\ 1 & 2 & -1 \end{bmatrix}$, if exist.
(b) Determine whether the vector $\vec{v} = (-5, 11, -7)$ is a linear combination of the vectors $\vec{v}_1 = (1, -2, 2)$, $\vec{v}_2 = (0, 5, 5)$ and $\vec{v}_3 = (2, 0, 8)$.
(c) When subjected to heat aluminium reacts with copper oxide to produce copper metal and aluminium oxide according to the equation
$$\text{Al}_3 + \text{CuO} \rightarrow \text{Al}_2\text{O}_3 + \text{Cu}$$
Using Gauss elimination method, balance the chemical equation.

Q.3
(a) Find the rank of a matrix $A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$.
(b) Solve the system of equation by Gauss-Jordan elimination method, if exist.
$$x + 4y - 3z = 0; \quad -x - 3y + 5z = -3; \quad 2x + 8y - 5z = 1.$$ 
(c) Let $V = \{(a, b): a, b \in \mathbb{R}\}$. Let $\overline{u} = (u_1, u_2)$ and $\overline{v} = (v_1, v_2)$. Define $(u_1, u_2) + (v_1, v_2) = (u_1 + v_1 + 1, u_2 + v_2 + 1)$ and $c(u_1, u_2) = (cu_1 + c - 1, cu_2 + c - 1)$. Verify that $V$ is a vector space.

Q.4
(a) Is $T: \mathbb{R}^2 \rightarrow \mathbb{R}$, defined by $T(x, y) = x^2 + y^2$ linear?
(b) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}$ be the mapping defined by $T(\overline{v}) = Av$ with $A = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$. Show that $T$ is one-to-one.
(c) Find the eigen values and eigen vectors of $A = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix}$.

Q.5
(a) If $\overline{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that $\nabla \text{logr} = \frac{1}{r}\hat{r}$. Where $\hat{r}$ is unit vector.
(b) Find the directional derivative of $\Phi = 4xz^3 - 3x^2y^2 z$ at the point $(2, -1, 2)$ in the direction $2\hat{i} + 3\hat{j} + 6\hat{k}$.
(c) Let $B$ be the basis for $\mathbb{R}^3$, given by $B = \{(1, 1, 1), (-1, 1, 0), (-1, 0, 1)\}$. Apply the Gram-Schmidt process to $B$ to find an orthonormal basis for $\mathbb{R}^3$.

Q.6
(a) Find constant $a, b, c$, so that $\overline{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational.
(b) Determine whether $V = \mathbb{R}^2$ is an inner product space under the inner product $\langle \overline{u}, \overline{v} \rangle = u_1^2v_1^2 + u_2^2v_2^2$.
(c) Verify the Green’s theorem in the plane for $\oint_C (y^2 dx + x^2 dy)$, where $C$ is the triangle bounded by $x = 0; x + y = 1$; and $y = 0$.

Q.7
(a) Determine whether $\overline{v}_1 = (2, 2, 2)$; $\overline{v}_2 = (0, 0, 3)$ and $\overline{v}_3 = (0, 1, 1)$ span the vector space $\mathbb{R}^3$.
(b) Consider the basis $S = \{\overline{v}_1, \overline{v}_2\}$ for $\mathbb{R}^2$, where $\overline{v}_1 = (-2, 1)$ and $\overline{v}_2 = (1, 3)$ and let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be the linear transformation such that $T(\overline{v}_1) = (-1, 2, 0)$ and $T(\overline{v}_2) = (0, -3, 5)$. Find a formula for $T(x_1, x_2)$.
(c) Verify rank nullity theorem for $A = \begin{bmatrix} 1 & 4 & 5 & 0 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix}$.

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