Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE SEMESTER– 1st /2nd (OLD SYLLABUS) EXAMINATION – SUMMER 2015

Subject Code: 110008 **Date: 02/06/2015**

Subject Name: Mathematics-1

Time: 10.30am-01.30pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Verify Cauchy's mean value theorem for $\frac{1}{x}$ and $\frac{1}{x^2}$ $\forall\ x\in[a,b]$, a>0. **03**

(ii) Find maxima and minima of the function f(x,y)=$x^3+y^3-3x-12y+20$. **04**

(b) (i) Find the Taylor's series expansion f(x,y)=x^3-2x+4 at a=2. **03**

(ii) Evaluate using L'Hospital rule **02**

$$\lim_{x\to 0}\ \frac{e^x-1-x}{x^2}$$

(iii) Evaluate using L'Hospital rule $\lim_{x\to 1}$ (1-x) $\tan(\frac{\pi x}{2})$ **02**

Q.2 **(a)** (i) Trace the curve $1\le r\le 2,\ 0\le\theta\le\frac{\pi}{2}$ **04**

(ii) Discuss convergence of the integral $\int_0^4 \frac{1}{x^{-2}}dx$. **03**

(b) (i) Evaluate $\int_{-\infty}^{\infty}\frac{dx}{1+x^2}$. **04**

(ii) State fundamental theorem of integral calculus and hence find $\frac{dy}{dx}$ if **03**

$y=\int_1^{x^2}\cos t\,dt$

Q.3 **(a)** (i) Test the convergence of $\sum_{n=1}^{\infty}\frac{3^n n!}{n^n}$. **04**

(ii) Does sequence whose n^{th} term is $a_n=\left(\frac{n+1}{n-1}\right)^n$ converges? If so find, limit of a_n. **03**

(b) (i) Show that the series $\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\ldots\ldots$ is convergent for p > 0. **04**

(ii) Find convergence of sequence $\left\{\frac{(-1)^n}{2}\right\}$. **03**

Q.4 **(a)** (i) State and prove Euler's theorem on homogenous functions. **04**

(ii) If $u = \text{cosec}^{-1}\left(\frac{x+y}{x^2+y^2}\right)$, show that $xu_x + yu_y = \tan u$. **03**

(b) (i) Find the equations of tangent plane and normal line to the surface $2x^2+y^2+2z=3$ at the point (2,1,-3). **04**

(ii) The region between the curve $y=\sqrt{x}$, $0 \le x \le 4$ and X-axis is revolved about X-axis is generate a solid then find volume. **03**

Q.5 **(a)** (i) Evaluate $\int_0^1 \int_y^{1+y^2} x^2 y\,dx\,dy$ **04**

(ii) If $x = r\cos\theta, y = r\sin\theta, z = z$, calculate $\frac{\partial(x,y,z)}{\partial(r,\theta,z)}$ **03**

(b) (i) Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2+y^2)\,dx\,dy$ **04**

(ii) If $u = 2xy, v = x^2 - y^2$, $x = r\cos\theta, y = r\sin\theta, z = z$, calculate $\frac{\partial(u,v)}{\partial(r,\theta)}$ **03**

Q.6 **(a)** (i) Find the direction derivative of $f(x,y,z) = 4xz^3-3x^2y^2z$ at the point (2,-1,2). **04**

(ii) Find the angle between the surfaces $x^2+y^2+z^2 = 9$ and $z = x^2+y^2-3$ at the point (2,-1,3). **03**

(b) (i) Evaluate using Green's theorem, $\int_c [(x^2+xy)dx + (x^2+y^2)dy]$ where c is the surface bounded by x = ±1, y = ±1. **04**

(ii) Prove that vector $\vec{F} = (z+\sin y)i + (x\cos y - z)j + (x-y)k$ is irrotational vector. **03**

Q.7 **(a)** (i) By changing the order of integration evaluate $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$. **04**

(ii) Find $\frac{dy}{dx}$ for $(\cos x)^y = x^{\sin y}$ using partial differentiation. **03**

(b) (i) Does limit of function $\frac{2xy}{x^2+3y^2}$ exists at x=0, y=0? **04**

(ii) Test the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n\log n}$. **03**

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