

**GUJARAT TECHNOLOGICAL UNIVERSITY**  
**BE-I - SEMESTER- 1<sup>st</sup> - EXAMINATION – SUMMER 2018**

Subject Code: 110008

Date: 21-05-2018

Subject Name: MATHS-I

Time: 02:30 pm to 05:30 pm

Total Marks: 70

Instructions:

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

**Q.1 (a)** (1) Find the value of  $k$  so that the function given below is continuous at  $x = 0$  **03**

$$f(x) = \begin{cases} \frac{\sin 3x}{2x} & x \neq 0 \\ k & x = 0 \end{cases}$$

(2) State Sandwich theorem on limit of sequences and using it find **04**

$$\lim_{x \rightarrow 0} g(x), \text{ if } 3 - x^3 \leq g(x) \leq 3 \sec x, \forall x \in R$$

**(b)** If  $u = f(r)$ , where  $r^2 = x^2 + y^2$ , prove that  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$  **07**

**Q.2 (a)** (1) Evaluate  $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$  **03**

(2) Find the area of the region between the x-axis and the graph of **04**

$$f(x) = x^3 - x^2 - 2x, \quad -1 \leq x \leq 2$$

**(b)** State (1) Rolle's Theorem **07**

(2) The Mean Value Theorem

Find the value of  $c$  using Mean Value Theorem, for the function

$$f(x) = 1 - x^2, \text{ in } 0 \leq x \leq 2$$

**Q.3 (a)** (1) Find the gradient of  $f(x, y, z) = 2z^3 - 3(x^2 + y^2)$  at the point (1,1,1). **03**

(2) Change the order of integration in the integral **04**

$$\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dy dx \quad \text{and evaluate it.}$$

**(b)** Trace the curve  $r = a(1 + \cos \theta)$ ;  $a > 0$  **07**

**Q.4 (a)** (1) Find the curl of  $\vec{F} = (x^2 - z)\hat{i} + xe^z\hat{j} + xy\hat{k}$  **03**

(2) If  $u = (x^2 + y^2 + z^2)^m$  then find  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$ . **04**

**(b)** Find the local extreme values of the function **07**

$$f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$$

**Q.5 (a) (1)** A fluid motion is given by **03**

$$\vec{v} = (y \sin z - \sin x) \hat{i} + (x \sin z + 2yz) \hat{j} + (xy \cos z + y^2) \hat{k}.$$

Show that the motion is irrotational.

(2) Expand  $\sin\left(\frac{\pi}{4} + x\right)$  in powers of  $x$ . Hence find the value of  $\sin 46^\circ$ . **04**

**(b)** Find the point on the plane  $x + 2y + 3z = 13$  closest to the point  $(1,1,1)$ . **07**

**Q.6 (a)** State modified Euler's Theorem. Show that, if  $u = \tan^{-1}\left(\frac{x^2 + y^2}{x - y}\right)$  **07**

$$x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = \frac{1}{4} \sin 4u - \frac{1}{2} \sin 2u$$

**(b)** State Green's Theorem. Using Green's theorem, evaluate **07**

$\oint_C e^{-x} (\sin y dx + \cos y dy)$  where  $C$  is the rectangle with vertices

$$(0,0), (\pi,0), (\pi, \pi/2), (0, \pi/2).$$

**Q.7 (a)** Write the statement of Cauchy's integral test. Test the convergence of the series **07**

$$\sum_{n=2}^{\infty} \frac{1}{n (\log n)^a}, \text{ for } 0 \leq a \leq 1.$$

**(b)** Find the volume of the solid of revolution of the area bounded by the curve **07**

$$y = xe^{-x} \text{ and the straight lines } x = 1, y = 0$$

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