Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

B.E. Sem-II [All Branch] examination June 2009

Subject code: 110009

Subject Name: Maths - II

Date: 17/06/2009 **Time: 10:30am-1:30pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **ATTEMPT THE FOLLOWING:**

(a) Define rank of the matrix. Find the rank of the matrix **03**

$$A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ -3 & 1 & 5 & 2 \\ -2 & 3 & 9 & 2 \end{bmatrix}$$

(b) Let R^3 have the Euclidean inner product. Use the Gram-Schmidt process to transform the basis vectors $u_1 = (1, 1, 1)$, $u_2 = (-1, 1, 0)$ and $u_3 = (1, 2, 1)$ into an orthonormal basis $\{v_1, v_2, v_3\}$. **04**

(c) Find the eigenvalues and bases for the eigenspaces of A^{25} and $A+2I$, where **03**

$$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$$

(d) i. Show that the functions $f(x) = x$ and $g(x) = \sin x$ form a linearly independent set of vectors in $C^1(-\infty, \infty)$. **01**

ii. Show that if $0 < \theta < \pi$, then $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ has no real eigenvalues and consequently no eigenvectors. **01**

iii. Define singular matrix. Find the inverse of the matrix A if it is invertible **02**

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

Q.2

(a) Determine the dimension and basis for the solution space of the system $3x_1 + x_2 + x_3 + x_4 = 0,\ 5x_1 - x_2 + x_3 - x_4 = 0$ **03**

(b) Justify your answer. Why the following sets are not vector space under the given operations? **02**

i. The set of all pairs of real numbers (x, y) with the operation $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$ and $\alpha(x, y) = (2\alpha x, 2\alpha y)$.

ii. In R^3, the operations defined as under $(x_1, y_1, z_1) + (x_2, y_2, z_2) = (z_1 + z_2, y_1 + y_2, x_1 + x_2)$

(c) Let λ be an eigenvalue of a matrix A. **02**
Then prove that (i) $\lambda + k$ is an eigenvalue of $A + kI$ (ii) $k\lambda$ is an eigenvalue of kA.

(d) i. Solve the following system by Gauss-Elimination method **04**

$2x+2y+2z=0,\ -2x+5y+2z=1,\ 8x+y+4z=-1$

ii. By using Gauss-Jordan elimination, Find the inverse of the given matrix **03**

$$A = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & \frac{-2}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{10} \\ \frac{1}{5} & \frac{-4}{5} & \frac{1}{10} \end{bmatrix}$$

OR

(d) I For which values of K and λ the following system have (i) no solution (ii)unique solution (iii) an infinite no. of solutions **04**

$x+y+z=6,\ x+2y+3z=10,\ x+2y+Kz=\lambda$

II Find basis for the row and column spaces of $A=\begin{bmatrix} 1 & 4 & 5 & 4 \\ 2 & 9 & 8 & 2 \\ 2 & 9 & 9 & 7 \\ -1 & -4 & -5 & -4 \end{bmatrix}$.

(b) Let $T:\mathbf{R}^2 \to \mathbf{R}^2$, defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ -x_2 \end{bmatrix}$ and let $B = \{e_1, e_2\}$ and $B' = \left\{\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \end{bmatrix}\right\}$. Then using $[T]_{B'} = P^{-1}[T]_B P$, find $[T]_{B'}$, where P is the transition matrix from B' to B. **03**

(c) Consider the basis $S = \{v_1, v_2\}$ of R^2, where $v_1 = (-2,1)$ and $v_2 = (1,3)$ and let $T:\mathbf{R}^2 \to \mathbf{R}^3$ be the linear transformation such that $T(v_1) = (-1,2,0)$ and $T(v_2) = (0,-3,5)$. Find a formula for $T(x_1, x_2)$, and use that to find $T(2,-3)$. **03**

(d) Define kernel of T. Let $T:\mathbf{R}^3 \to P_1$ be a linear transformation defined by $T(a_1,a_2,a_3) = (-a_1 + 2a_2 + a_3) + (-a_2 + a_3)x$. Find which of the following vectors are in $ker(T)$; $(i)\, u = (6,2,2), (ii)\, u = (2,-1,1)$ and $(iii)\, u = (0,0,0)$. **02**

(e) If u and v are orthogonal unit vectors, then what is the distance between u and v? Justify your answer. **02**

(f) Define: Algebraic multiplicity of an eigenvalue. Determine the algebraic and geometric multiplicity of $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ if the eigenvalues of A are $\lambda = 2,-1,-1$ and corresponding eigenvectors for $\lambda = 2$ is $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and for $\lambda = -1$ are $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$. **02**

OR

Q.4 (a) State Cauchy-Schwarz inequality. Verify Cauchy-Schwarz inequality for the vectors $u = (-3,1,0), v = (2,-1,3)$. **02**

(b) Prove that $< u,v,> = \frac{1}{4}\| u+v \|^2 - \frac{1}{4}\| u-v \|^2$ **01**

(c) Let $T: R^4 \to R^3$ be the linear transformation given by the formula $T(x_1,x_2,x_3,x_4) = (4x_1 + x_2 - 2x_3 - 3x_4, 2x_1 + x_2 + x_3 - 4x_4, 6x_1 - 9x_3 + 9x_4)$. Find a basis and dimension of $\ker(T)$, $rank(T)$ and verify the dimension theorem. **05**

(d) Find the projection of $u = (1,-2,3)$ along $v = (1,2,1)$ in $\mathbf{R}^3$. **01**

(e) Let $S,T:\mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformations given by the formulas $T(x,y) = (x+y, x-y)$ and $S(x,y) = (2x+y, x-2y)$. (i) Show that S and T are one to one, (ii) Find formula for $T^{-1}(x,y)$, $S^{-1}(x,y)$ and $(S\circ T)^{-1}(x,y)$, (iii) Verify that $(S\circ T)^{-1} = T^{-1}\circ S^{-1}$. **05**

Q.5 (a) Find a matrix P that diagonalizes $A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$, and determine $P^{-1}AP$. **06**

(b) By using Cayley-Hamilton theorem, if $A=\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$, then prove that **04**

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I = \begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$$

(c) Find a change of variables that reduces the quadratic form $2x_1^2 + 2x_2^2 - 2x_1x_2$ to a sum of squares and express the quadratic form in terms of the new variables. **04**

OR

Q.5 (a) Find an orthogonal matrix P that diagonalizes $A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$. **06**

(b) Find the least squares solution of the linear system $Ax = b$ given by $x_1 + x_2 = 7, -x_1 + x_2 = 0, -x_1 + 2x_2 = -7$ and find the orthogonal projection of b on the column space of A. **04**

(c) Describe the conic whose equation is $5x^2 - 4xy + 8y^2 - 36 = 0$. **04**

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