Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

B.E Sem-I/II Examination June-July 2011

Subject code: 110009 **Subject Name: Maths-II**

Date: 8/7/2011 **Total Marks: 70** **Time: 10:30 am to 1:30pm**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Use Gauss-Jordan Method to find A^{-1}, if **05**

$$A = \begin{bmatrix} 7 & 6 & 2 \\ -1 & 2 & 4 \\ 3 & 6 & 8 \end{bmatrix}.$$

(ii) Show that the inner product **02**

$$\langle u, v\rangle = 13u_1v_1 - 5u_1v_2 - 5u_2v_1 + 17\, u_2v_2$$

is generated by the matrix $A = \begin{bmatrix} 3 & 1 \\ 2 & -4 \end{bmatrix}$.

(b) (i) Test for consistency and solve **05**

$$5x + 3y + 7z = 4$$
$$3x + 26y + 2z = 9$$
$$7x + 2y + 10z = 5$$

(ii) Find the matrix representation of the quadratic form **02**

$$x_1^2 + 7x_2^2 - 3x_3^2 + 4x_1x_2 - 2x_1x_3 + 6x_2x_3 .$$

Q.2 **(a)** Define rank of a matrix. Determine the rank of the matrix A, if A **07**

$$= \begin{bmatrix} 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \\ 10 & 11 & 12 & 13 & 14 \\ 15 & 16 & 17 & 18 & 19 \end{bmatrix}.$$

(b) Define adjoint of a matrix. Find the adjoint and inverse of A by determent method: **07**

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}.$$

OR

(b) (i) Show that the set of vectors {(2, 1, 1), (1, 2, 2), (1, 1, 1)} is linearly dependent in R^3. **04**

(ii) Prove that the set of vectors {(1, 2, 2), (2, 1, 2), (2, 2, 1)} is linearly independent in R^3. **03**

Q.3 **(a)** (i) State Cauchy-Schwarz inequality in R^n. Verify Cauchy-Schwarz inequality for the vectors $u_1 = (0, -2, 2, 1)$, $u_2 = (-1, -1, 1, 1)$. **03**

(ii) Define linear combination of vectors $v_1, v_2, v_3, \ldots, v_n$. Express a vector $v = (7, 4, -3)$ as a linear combination of $v_1 = (1, -2, -5)$, $v_2 = (2, 5, 6)$. **04**

(b) (i) If $S = \{v_1, v_2, v_3, v_4\}$ be a subset of a vector space V. Define span S. Determine whether $v_1 = (1, 1, 2)$, $v_2 = (1, 0, 1)$ and $v_3 = (2, 1, 3)$ span the vector space R^3. **03**

(ii) Define subspace of a vector space. Check whether **04**

1. $W = \{(x, y) \in R^2 / x \geq 0, y \geq 0\}$ is a subspace R^2.
2. $W = \{(x, y, z) \in R^3 / ax + by + cz = 0, a, b, c \in R\}$ is a subspace R^3.

OR

Q.3 **(a)** Define basis of a vector space. If $v_1 = (1, 2, 1)$, $v_2 = (2, 9, 0)$, $v_3 = (3, 3, 4)$, show that $S = \{v_1, v_2, v_3\}$ is a basis for R^3. Find the coordinates of $v = (5, -1, 9)$ with respect to S. **07**

(b) Define row space, column space and null space of a matrix. Find the row space and column space of the matrix **07**

$$A = \begin{bmatrix} 2 & -3 & 6 \\ 1 & -1 & 5 \\ -1 & 2 & 0 \\ 4 & 1 & 1 \end{bmatrix}$$

Q.4 **(a)** Define linear transformation in Euclidean vector space. Check the linearity of the following transformation. **07**

(i) $T: R^3 \to R^3$ defined by $T(x_1, x_2, x_3) = (x_1 + 1, x_2 + 2, x_3 + 4)$

(ii) $T: M_{nn} \to R$ defined by $T(A) = \det A$.

(b) Find the associated matrix of the linear transformation $T: R^3 \to R^3$, $T(u_1, u_2, u_3) = (u_1 + u_2, u_2 + u_3, u_3 + u_1)$ with basis $B = \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$ and $B' = \{(1, 0, 1), (0, 1, 0), (1, 0, -1)\}$ for the domain and co-domain of T respectively. **07**

OR

Q.4 **(a)** If $T: P_2 \to P_3$ is the linear transformation defined by $T(p(x)) = xp(x+1)$, then find the matrix $[T]_{B',B}$ with respect to the basis $B = \{1, x, x^2\}$ and $B' = \{1, x, x^2, x^3\}$. **07**

Q.4 **(b)** State rank-nullity theorem. Find the rank and nullity of transformation $T: R^3 \to R^2$, $T(x, y, z) = (x + y + z, x + y)$. **07**

Q.5 **(a)** Show that the set V = {(1, 0, 0), (0, 1, 0), (0, 0, 1)} is an orthonormal set in R^3 with Euclidean inner product. **07**

(b) Find the Eigen values and Eigen vectors of the matrix **07**

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

OR

Q.5 **(a)** Find a matrix that diagonalizes A and determine $P^{-1}AP$, where $A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. **07**

(b) Let R^3 have the Euclidean inner product. Use the Gram-Schmidt process to transform the basis $\{u_1, u_2, u_3\}$ into an orthonormal basis where $u_1 = (1, 1, 1)$, $u_2 = (-1, 1, 0)$ and $u_3 = (1, 2, 1)$. **07**

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