Seat No.: ________ Enrolment No.______________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- Ist /IInd SEMESTER–EXAMINATION – MAY/JUNE - 2012

Subject code: 110009 **Date: 26/05/2012**

Subject Name: Mathematics - II

Time: 10:30 am – 01:30 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Each question carry equal marks**

Q.1	(a)	Solve by Gauss Jordan Method $x+y+z=3, x+2y-z=4, x+3y+2z=4$	**3**
	(b)	Express the matrix $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ as a product of Elementary Matrix.	**7**
	(c)	Find the inverse of the matrix $A=\begin{bmatrix}1 & -1 & 1\\ 0 & 1 & 2\\ 1 & 2 & 2\end{bmatrix}$	**4**
Q.2	(a)	Prove that $\left\lVert \bar{u}+\bar{v}\right\rVert^2+\left\lVert \bar{u}-\bar{v}\right\rVert^2=2\left\lVert \bar{u}\right\rVert^2+2\left\lVert \bar{v}\right\rVert^2$	**3**
	(b)1	Let $\bar{u}=(2,4,6)$and $\bar{a}=(1,-1,-1)$. Find the vector component of $\bar{u}$ along $\bar{a}$ and orthogonal to $\bar{a}$.	**4**
	2	Give a 3×3 singular matrix such that all the entries in the matrix are non zero.	**2**
	(c)1	Consider $\bar{u}=(6,2,3), \bar{v}=(2,2,-1)$ and $\bar{w}=(1,0,1)$ with same initial point. Determine whether they are in the same plane?	**2**
	2	State Cauchy Schwarz inequality in R^n and verify for the vectors $\bar{u}=(1,2,-3)$ and $\bar{v}=(1,0,2)$.	**3**
Q.3	(a)1	Write the standard matrix for linear transformation $T(x,y,z,w)=(3x+z+w,\ x-y-w)$	**2**
	2	Use matrix multiplication to find the reflection of $(2,-1,4)$ about xy - plane.	**2**
	3	Use matrix multiplication to find the image of the vector $(1,-1,3)$ if it is rotated -30^0 about the x – axis.	**2**
	(b)	Show that $T:R^2\to R^2$ defined by the equation $w_1=x_1+2x_2,\quad w_2=2x_1+3x_2$ Is one – to – one and find $T^{-1}(w_1,w_2)$.	**5**
	(c)	Is $T(x,y)=(x+3y,\ y-2)$ linear operator?	**3**
Q.4	(a)	Consider real vector space P_2 with standard operations. Is $S_1=\{a_0+a_1x+a_2x^2 / a_0=0\}$ and $S_2=\{a_0+a_1x+a_2x^2 / a_0\neq0\}$ are subspaces? Give reasons.	**4**

	(b)	Find an equation of the plane spanned by the vectors $\overline{u}=(1,1,2),\ \overline{v}=(3,2,0)$	**4**
	(c)1	Is the set $S=\{3+x+x^2,\ 2-x+5x^2, 4-3x^2\}$ linearly independent in P_2 with standard operations?	**3**
	2	Write any three bases of P_2.	**3**
Q.5	(a)	Determine the basis of plane $x+4y-3z=0$.	**4**
	(b)	Let $S=\{\overline{v}_1,\overline{v}_2,\overline{v}_3,\overline{v}_4,\overline{v}_5\}$. Find a subset of S that forms a basis for the space spanned by S. Also express other vectors as linear combination of basis vectors. Here, $\overline{v}_1=(1,-1,2), \overline{v}_2=(0,1,1), \overline{v}_3=(1,2,8), \overline{v}_4=(0,3,6), \overline{v}_5=(-1,3,3)$	**4**
	(c)1	Find a basis for the null space of $A=\begin{bmatrix}1&0&3\\-2&1&0\\-1&1&3\end{bmatrix}$.	**3**
	2	Find the rank and nullity of the matrix $A=\begin{bmatrix}1&1&2&0\\0&-1&1&1\\2&0&3&1\end{bmatrix}$	**3**
Q. 6	(a)	Define an inner product on real vector space V. Let $\overline{u}=(u_1,u_2),\quad \overline{v}=(v_1,v_2)$ are any vectors in R^2. Show that $<\overline{u},\overline{v}>=9u_1v_1+4u_2v_2$ is inner product on R^2.	**4**
	(b)	Find a basis for the orthogonal complement of $W=\{\overline{v}_1,\overline{v}_2,\overline{v}_3,\overline{v}_4\}$, where $\overline{v}_1=(1,-1,1,0,2)$, $\overline{v}_2=(0,3,1,0,0)$, $\overline{v}_3=(-1,0,0,1,-1)$, $\overline{v}_4=(0,-1,1,1,1)$.	**4**
	(c)1	Apply Gram – Schmidt process to transform the basis vectors $\overline{u}_1=(1,1,1), \overline{u}_2=(0,1,1), \overline{u}_3=(0,0,1)$ in to an orthogonal basis.	**4**
	2	Write any three orthogonal matrices of the type 2×2.	**2**
Q.7	(a)	Find eigen basis of the matric $A=\begin{bmatrix}0&0&1\\0&2&0\\3&0&0\end{bmatrix}$	**5**
	(b)	Find a matrix P that diagonalizes $A=\begin{bmatrix}1&0\\6&-1\end{bmatrix}$.	**4**
	(c)1	Is the function $T:P_2\to P_2,\quad T(a_0+a_1x+a_2x^2)=a_0+a_1(x+2)+(a_2+2)x^2$ linear transformation?	**3**
	2	Let $T:R^2\to R^2$, $T(x,y)=(x+y,x-y)$. Determine T^{-1} if possible.	**2**

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