Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER– 1st / 2nd • EXAMINATION – SUMMER 2016

Subject Code: 110009 **Date:30/05/2016**

Subject Name: MATHS-II

Time: 02:30 PM to 05:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Solve the following system of linear equation by using Gauss elimination **07**
$x+y+2z=9$; $2x+4y-3z=1$; $3x+6y-5z=0$

(b) (i) Find the inverse of a matrix $A=\begin{bmatrix}1&2&1\\0&2&2\\2&1&1\end{bmatrix}$ using row operation. **04**

(ii) Check whether the set $W=\{a_0+a_1x+a_2x^2+a_3x^3 / a_0=0\}$ is a subspace of P_3. **03**

Q.2 **(a)** Find the rank and nullity of the matrix $A=\begin{bmatrix}3&4&5&6&7\\4&5&6&7&8\\5&6&7&8&9\\10&11&12&13&14\\15&16&17&18&19\end{bmatrix}$ **07**

(b) (i) Define orthogonal matrix and verify $A=\frac{1}{3}\begin{bmatrix}1&2&2\\2&1&-2\\-2&2&-1\end{bmatrix}$ is an orthogonal matrix. **04**

(ii) Check whether the given vectors $v_1=(1,-1,1)$, $v_2=(0,1,2)$, $v_3=(3,0,-1)$ forms a basis of R^3 **03**

Q.3 **(a)** Determine whether the following functions are linear transformation. Justify your answer. **07**
(i) Let $T:R^2 \to R^2$ *where* $T(x,y)=(x+2y,3x-y)$
(ii) Let $T:M_{nn} \to R$ *where* $T(A)=\det(A)$

(b) (i) Find the transition matrix from basis $B=\{(1,0),(0,1)\}$ of R^2 to basis $B'=\{(1,1),(2,1)\}$ of R^2. **04**

(ii) Determine whether matrix $A=\begin{bmatrix}1&0&1\\0&1&1\\1&1&0\end{bmatrix}$ is one-to-one and onto. **03**

Q.4 **(a)** Prove that the set of all positive real numbers forms a vector space under the operations defined by vector addition : $x+y=x\cdot y$ and scalar multiplication : $\alpha x=x^{\alpha}$ for all $x,y\in R^{+}$. **07**

(b) (i) Determine whether the following polynomials span P_2: **04**

$P_1 = 1 - x + 2x^2$; $P_2 = 5 - x + 4x^2$ $P_3 = -2 - 2x + 2x^2$

(ii) Show that $f_1 = 1, f_2 = e^x, f_3 = e^{2x}$ form a linearly independent set of vectors in $C^2(-\infty, \infty)$. **03**

Q.5 **(a)** Consider the basis $S = \{v_1, v_2\}$ for R^2, where $v_1 = (-2,1)$ *and* $v_2 = (1,3)$ and Let $T: R^2 \to R^3$ be the linear transformation such that $T(v_1) = (-1,2,0)$ and $T(v_2) = (0,-3,5)$. Find a formula for $T(x_1, x_2)$ and use that formula to find $T(2,3)$. **07**

(b) (i) Reduce the quadratic form $Q(x,y) = x_1^2 + 4x_2^2 + x_3^2 - 4x_1x_2 + 2x_3x_1 - 4x_2x_3$ into canonical and find nature and signature. **04**

(ii) Is $A = \begin{bmatrix} 0 & 2-3i & 1+i \\ -2-3i & 2i & 2-i \\ -1+i & -2-i & -i \end{bmatrix}$ a skew Hermition matrix? **03**

Q.6 **(a)** Verify that the basis vectors $v_1 = \left(\frac{-3}{5}, \frac{4}{5}, 0\right)$, $v_2 = \left(\frac{4}{5}, \frac{3}{5}, 0\right)$ *and* $v_3 = (0,0,1)$ form an orthonormal basis S for R^3 with the Euclidean inner product. Express the vector $u = (1,-1,2)$ as a linear combination of the vectors v_1, v_2, v_3 and find coordinate vector $[u]_s$. **07**

(b) (i) Find the orthogonal projection of $u = (1,-2,3)$ and $v = (1,2,1)$ in R^3 with respect to
the Euclidean inner product. **04**

(ii) Find $\langle f, g \rangle = \int_{-1}^{1} f(x)g(x)dx$ if $f(x) = 1 - x + x^2 + 5x^3$ *and* $g(x) = x - 3x^2$ **03**

Q.7 **(a)** Find a matrix P that diagonalize the matrix $A = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$ hence find A^{10}. **07**

(b) (i) Determine algebraic and geometric multiplicity of each eigen value of the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$. **04**

(ii) If $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ then find the eigenvalues of A^2 *and* A^{-1}. **03**

****