Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER– 1st / 2nd (OLD) EXAMINATION – SUMMER 2018

Subject Code: 110009 **Date: 17-05-2018**

Subject Name: MATHEMATICS-II

Time: 02:30 pm to 05:30 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) For which values of k , u and v orthogonal? **03**

$u = (2,1,3), v = (1,7,k)$

(ii) Verify Cauchy-Schwarz inequality for the vectors **04**

$u = (0,-2,2,1), v = (-1,-1,1,1)$

(b) (i) Find the rank for the matrix $\begin{bmatrix} 1 & 6 & 8 \\ 2 & 5 & 3 \\ 7 & 9 & 4 \end{bmatrix}$. **03**

(ii) Solve the following linear system by using Gauss Jordan method. **04**

$$3x - 2y + 8z = 9$$
$$-2x + 2y + z = 3$$
$$x + 2y - 3z = 8$$

Q.2 **(a)** (i) Solve the following linear system by using Gauss Elimination method **03**

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + 4z = 1$$

(ii) Find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. **04**

(b) (i) Prove that the matrix $A = \begin{bmatrix} -1 & 2+i & 5-3i \\ 2-i & 7 & 5i \\ 5+3i & -5i & 2 \end{bmatrix}$ is a hermitian matrix. **03**

(ii) Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ **04**

Q.3 **(a)** Show that the set of all 2×2 matrices of the form $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$ with addition defined by $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} + \begin{bmatrix} c & 1 \\ 1 & d \end{bmatrix} = \begin{bmatrix} a+c & 1 \\ 1 & b+d \end{bmatrix}$ and scalar multiplication defined by $k\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} = \begin{bmatrix} ka & 1 \\ 1 & kb \end{bmatrix}$ is a vector space. **07**

(b) (i) Determine whether the vectors $(1,2,3),(3,-2,1)$ and $(1,-6,-5)$ are linearly dependent or linearly independent. **03**

(ii) Determine whether the vectors $(1,-1,1),(0,1,2),(3,0,-1)$ forms basis for R^3 **04**

Q.4 **(a)** Extend the subset $A=\{(1,-2,5,-3),(2,3,1,-4)\}$ of R^4 to the basis for vector space R^4. **07**

(b) (i) Find two vector in R^2 with Euclidean norm whose inner product with $(-3,1)$ is zero. **03**

(ii) Obtain the matrix of a linear transformation $T:R^3\to R^3$ defined by $T(x,y,z)=(2x,x+y+z,x+3z)$ with respect to the basis $B_1=\{(1,0,0),(1,1,0),(1,1,1)\}$ and $B_2=\{(1,0,0),(0,1,0),(0,0,1)\}$. **04**

Q.5 **(a)** For the basis $S=\{u,v,w\}$ of R^3, where $u=(1,1,1), v=(1,1,0)$ and $w=(1,0,0)$, let $T:R^3\to R^3$ be a linear transformation such that $T(u)=(2,-1,4), T(v)=(3,0,1), T(w)=(-1,5,1)$. Find a formula for $T(x,y,z)$ and use it to find $T(2,4,-1)$. **07**

(b) State Rank-Nullity theorem. **07**

Let $T:R^4\to R^3$ be a linear transformation defined by $T(1,0,0,0)=(1,1,1), T(0,1,0,0)=(1,-1,1), T(0,0,1,0)=(1,0,0), T(0,0,0,1)=(1,0,1)$. Then verify the rank-nullity theorem.

Q.6 **(a)** Find the least square solution of the linear system $AX=b$ given by **07**

$$x_1+x_2=7$$
$$-x_1+x_2=0$$
$$x_1+2x_2=-7$$

(b) Let R^3 have the Euclidean inner product. Use the Gram Schmidt process to transform the basis $\{u_1,u_2,u_3\}$ into an orthonormal basis, where $u_1=(1,0,0), u_2=(3,7,-2), u_3=(0,4,1)$ **07**

Q.7 **(a)** Find a matrix that diagonalizes and determine $P^{-1}AP$, where $A=\begin{bmatrix}2 & 0 & -2\\ 0 & 3 & 0\\ 0 & 0 & 3\end{bmatrix}$. **07**

(b) (i) Find the algebraic and geometric multiplicity of $A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & -3 & 3\end{bmatrix}$. **03**

(ii)Verify Caley-Hamilton theorem for the matrix, $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$. **04**

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