Seat No.: ________ Enrolment No.______________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- Ist /IInd SEMESTER–EXAMINATION – MAY/JUNE - 2012

Subject code: 110014 **Date: 06/06/2012**

Subject Name: Calculus

Time: 10:30 am – 01:30 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Each question carry equal marks**

Q. 1. (a) (i) Find the values of $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial x\, \partial y}$ at the point (1 , 2) for $f(x, y) = x^2 + 3xy + y - 1$ **[02]**

(ii) If $u = x^2y + y^2z + z^2x$, find the values of **[04]**

(1) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$

(2) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$

(b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, show that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2 \sin u \cos 3u$ **[04]**

(c) If $u = f(l, m)$ where $l = x + y$ and $m = x - y$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 2\frac{\partial u}{\partial l}$ **[04]**

Q. 2. (a) (i) Find the equation of the tangent plane to the surface $x^2 + 2y^2 + 3z^2 = 12$ at (1, 2, –1) **[02]**

(ii) Using reduction formula, evaluate $\int_0^{\pi} x \sin^5 x \cos^4 x \, dx$ **[04]**

(b) Using partial derivatives, find $\frac{d^2 y}{dx^2}$ for $x^5 + y^5 = 5x^2$ **[04]**

(c) Using appropriate reduction formulae, evaluate

(i) $\int_0^{\pi} \sin^2 x \, (1 + \cos x)^4 \, dx$ **[02]**

(ii) $\int_0^{\infty} \frac{dx}{(1 + x^2)^{3/2}}$ **[02]**

Q. 3. (a) (i) Show that the sequence $\left\{\frac{3}{n+3}\right\}$ is a decreasing sequence. **[02]**

(ii) Using comparison test, discuss the convergence of

(1) $\sum \frac{\sqrt{n}-1}{n^2+1}$ **[02]**

(2) $\sum_{n=1}^{\infty} \frac{1}{n}\sin\left(\frac{1}{n}\right)$ **[02]**

(b) Test the convergence of the series **[04]**

$\sqrt{\frac{1}{2}}\ x+\sqrt{\frac{2}{5}}\ x^2+\sqrt{\frac{3}{10}}\ x^3+\ldots.\infty\ ,\quad x>0$

(c) Discuss the convergence of the series $\sum_{n=1}^{\infty}\left(\frac{n+1}{n+2}\right)^n x^n$ where $x>0$ **[04]**

Q. 4. (a) (i) Using Maclaurin's Series, prove that $\tan h^{-1}x = x+\frac{x^3}{3}+\frac{x^5}{5}+\ldots$ **[02]**

(ii) Expand the function $f(x) = x^4 - 11x^3 + 43x^2 - 60x + 14$ in powers of $(x-3)$. **[04]**

(b) Expand $\frac{e^x}{\cos x}$ upto first four terms by Maclaurin's Series. **[04]**

(c) Evaluate: $\lim_{x\to 0} \frac{(1+x)^{1/x}-e+\frac{1}{2}ex}{x^2}$ **[04]**

Q. 5. (a) (i) Evaluate: $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dy\,dx}{1+x^2+y^2}$ **[02]**

(ii)Evaluate $\iint_R xy\ dxdy$ where the region R is bounded by $x^2+y^2-2x=0$, $y^2=2x$, $y=x$ **[04]**

(b) Evaluate: $\iint_R \frac{r\,dr\,d\theta}{\sqrt{a^2+r^2}}$, where R is a loop of $r^2 = a^2\cos 2\theta$ **[04]**

(c) Evaluate: $\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} dy\,dx$ by changing the order of integration. **[04]**

Q. 6. (a) (i) Evaluate: $\int_{1}^{3}\int_{1/x}^{1}\int_{0}^{\sqrt{xy}} xyz\,dz\,dy\,dx$ **[02]**

(ii) Find by double integration the area of the region that lies inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$ **[04]**

(b) Using triple integration, find the volume of the tetrahedron bounded by the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c}$ and the co-ordinate plane. **[04]**

(c) Find a point on the plane $2x + 3y - z = 5$ which is nearest to the origin, using Lagrange's method of undetermined multipliers. **[04]**

Q. 7. (a) Answer the following :

(1) Find $\lim_{n\to\infty} \frac{\sin n}{n}$ **[02]**

(2) Find the points of inflection on the curve $f(x) = (x + 2)^3$ **[02]**

(b) (1) Find the tangents at the origin to the curve $y^2(a + x) = x^2(3a - x)$ **[02]**

(2) Find the cusp for the curve $a^2x^2 = y^3(2a - y)$ **[02]**

(c) (1) Find $\frac{d}{dx}\left[\int_{1}^{x^2} \cos t\ dt\right]$ **[03]**

(2) Find the total area bounded by $a^2y^2 = x^2(a^2 - x^2)$ **[03]**

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