

**GUJARAT TECHNOLOGICAL UNIVERSITY****BE SEMESTER– 1<sup>st</sup>/2<sup>nd</sup> (OLD SYLLABUS) EXAMINATION – SUMMER 2015****Subject code: 110014****Date: 02/06/2015****Subject Name: Calculus****Time: 10.30am-01.30pm****Total Marks: 70****Instructions:**

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Each question carries equal marks.

**Q.1 (a)** Do as Directed:

- (i) State Reduction formula for  $\int_0^{\pi/2} \sin^n x dx$ , ( $n \in N$ ). **03**

Hence, evaluate  $\int_0^{\pi} x \sin^5 x dx$ .

- (ii) Expand  $f(x) = e^x \cos x$  in powers of  $x$  up to the terms containing  $x^4$ . **04**

**(b)** Do as Directed:

- (i) Determine whether  $\int_0^{\pi/2} \sec x dx$  converges or diverges. **03**

- (ii) Evaluate the limits: **04**

$$(i) \lim_{x \rightarrow 0} \frac{x(\cos x - 1)}{\sin x - x}; \quad (ii) \lim_{x \rightarrow 0} \frac{2\sqrt{1+x} - 2 - x}{2\sin^2 x}.$$

**Q.2 (a)** Do as Directed:

- (i) Find the critical points of  $f(x) = 2x^3 - 14x^2 + 22x - 5$ . Also, find the points of inflection. **03**

- (ii) Using double integrations, find the area of the region common to the parabolas  $y^2 = 8x$  and  $x^2 = 8y$ . **04**

**(b)** Do as Directed:

- (i) Test for convergence or divergence:  $\sum_{n=1}^{\infty} \frac{n^{10}}{10^n}$ . **03**

- (ii) Change into polar coordinates and hence evaluate:  $\int_0^a \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} y^2 dy dx$ . **04**

**Q.3 (a)** Do as Directed:

- (i) Find the local extreme values of the function  $f(x, y) = x^3 + y^3 - 3xy$ . **03**

- (ii) Find the Maclurin's series expansion of  $f(x, y) = \sin(2x + 3y)$  up to third order derivative terms. **04**

**(b)** Do as Directed:

- (i) State Euler's theorem for function of two variables and apply it to find  $x^2 f_{xx} + 2xy f_{yx} + y^2 f_{yy}$  for  $f(x, y) = \frac{x^5 - y^5}{x^2 + 3xy}$ . **03**

- (ii) Evaluate  $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} z dz dy dx$ . **04**

- Q.4 (a)** Do as Directed: **03**
- (i) Evaluate  $\int_0^{2a} x^{5/2} \sqrt{2ax - x^2} dx$ . **03**
- (ii) If  $x = r \sin \theta \cos \phi$ ;  $y = r \sin \theta \sin \phi$ ;  $z = r \cos \theta$  then find  $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$ . **04**
- (b)** Do as Directed:
- (i) Find the equations of tangent plane and normal line of the surface  $x^2 + y^2 - z^2 = 4$  at the point  $(1, 2, 1)$ . **03**
- (ii) Change into spherical polar coordinates and hence evaluate **04**
- $$\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{dz dy dx}{\sqrt{a^2-x^2-y^2-z^2}}.$$
- Q.5 (a)** Do as Directed:
- (i) Find the area of the loop of the curve  $9ay^2 = x^2(3a-x)$ . **03**
- (ii) If  $z = f(x, y)$ , where  $x = s+t$ ;  $y = s-t$ , show that **04**
- $$\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = \frac{\partial z}{\partial s} \cdot \frac{\partial z}{\partial t}.$$
- (b)** Do as Directed:
- (i) State Leibniz's rule and hence evaluate  $\frac{d}{dx} \left[ \int_{x^2}^{\sqrt{x}} \cos(t+1) dt \right]$ . **03**
- (ii) If  $z = \sin^{-1}(x-y)$ , where  $x = 3t$ ;  $y = 4t^3$ , show that  $\frac{dz}{dt} = 3(1-t^2)^{-\frac{1}{2}}$ . **04**
- Q.6 (a)** Do as Directed:
- (i) If  $f(x, y) = \frac{x^2 y}{x^4 + y^2}$ , does  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$  exist? **03**
- (ii) Find by double integrations, the volume of the cylinder  $x^2 + y^2 = 1$  between the planes  $z = 0$  and  $y + z = 2$ . **04**
- (b)** Do as Directed:
- (i) Find the asymptotes parallel to the coordinate axes of the curve  $(x^2 - 4)(y^2 - 9) = 36$ . **03**
- (ii) Find the volume by triple integrals of the solid S that is bounded by the plane  $2x + 3y + 4z = 12$  and the three coordinate planes. **04**
- Q.7 (a)** Do as Directed:
- (i) Discuss the symmetry of the curve  $y^2(a^2 - x^2) = x^2(a^2 + x^2)$ . **03**
- (ii) Change the order of Integrations and hence evaluate:  $\int_0^1 \int_{2x}^2 e^{y^2} dx dy$ . **04**
- (b)** Do as Directed:
- (i) Using double integrations, find the area enclosed by the cardioid  $r = a(1 + \cos \theta)$ . **03**
- (ii) Find the volume of the solid obtained by rotating the region bounded by  $y = x^3$ ,  $y = 8$  and  $x = 0$  about the  $y$ -axis. **04**

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