

**GUJARAT TECHNOLOGICAL UNIVERSITY**  
**BE- SEMESTER 1<sup>st</sup> / 2<sup>nd</sup> EXAMINATION (OLD SYLLABUS) – SUMMER - 2017**

**Subject Code: 110014****Date: 01/06/2017****Subject Name: Calculus****Time: 2:30 PM to 05:30 PM****Total Marks: 70****Instructions:**

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1 (a)** 1) Find the Taylor's series about  $x = 0$  for  $f(x) = \frac{1}{1+x^2}$ . **03**
- 2) Find the expansion of  $\tan\left(x + \frac{\pi}{4}\right)$  in ascending powers of  $x$  up to terms in  $x^3$  and find approximately the value of  $\tan 43^\circ$ . **04**
- (b)** 1) Does the series  $\sum_{n=1}^{\infty} (1 - e^{-n})$  converges? Justify. **03**
- 2) Evaluate: **04**
- i.  $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$
- ii.  $\lim_{x \rightarrow 0} (\sin x)^{\tan x}$
- Q.2 (a)** 1) Determine whether the series  $\sum_{n=1}^{\infty} \frac{(n-1)!}{n^2}$  converges or diverges. **03**
- 2) Find the radius and interval of convergence of the series  $\sum_{n=1}^{\infty} (-1)^n \frac{x^{2n-1}}{(2n-1)!}$ . **04**
- (b)** 1) Determine whether the series  $\sum_{n=1}^{\infty} \frac{(n-1)}{n^3+3}$  converges or diverges. **03**
- 2) Discuss the continuity of **04**
- $$f(x, y) = \begin{cases} \frac{x^2 - y^2}{x^2 + y^2} & ; (x, y) \neq (0, 0) \\ 0 & ; (x, y) = (0, 0) \end{cases} \text{ at } (0, 0).$$
- Q.3 (a)** 1) Does the improper integral  $\int_0^{\infty} e^{-5x} dx$  converge or diverge? Justify. **03**
- 2) Compute the four second order partial derivatives of  $f(x, y) = xy^2 + 3x^2e^y$ . **04**
- (b)** 1) Evaluate  $\lim_{x \rightarrow 0} \left( \frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$ . **03**
- 2) Find the maximum and minimum values of  $x + y$  on the circle  $x^2 + y^2 = 4$ . **04**
- Q.4 (a)** 1) Find the equation of the tangent plane to the sphere  $x^2 + y^2 + z^2 = 14$  at the point  $(1, 2, 3)$ . **03**
- 2) If  $u = \cos^{-1} \frac{x^3 + y^3}{x + y}$ , show that **04**
- $$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\sin 2u \sin u - 4 \cos u}{\sin^3 u}.$$
- (b)** 1) Determine the intervals of increasing and decreasing for the function  $f(x) = x^3 - 12x - 5$ . **03**
- 2) Evaluate  $\int \int_D (x + 2y) dA$ , where  $D$  is the region bounded by the parabolas  $y = 2x^2$  and  $y = 1 + x^2$ . **04**

- Q.5 (a)** 1) The region between the curve  $y = \sqrt{x}$ ;  $0 \leq x \leq 4$  and the  $x$ -axis is revolved about the  $x$ -axis to generate a solid. Find its volume. **03**
- 2) Find the maximum and minimum values of the function  $f(x) = x^3 + y^3 - 3x - 12y + 20$ . **04**
- (b)** 1) Evaluate  $\int_0^1 \int_{x^2}^x (1 + xy) dy dx$  **03**
- 2) Evaluate the integral  $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx$  by reversing the order of integration. **04**
- Q.6 (a)** Trace the curve  $xy^2 = 4a^2(2a - x)$ ;  $a > 0$ . **07**
- (b)** 1) Evaluate the improper integral  $\int_1^\infty \frac{1}{x^2} dx$ . **03**
- 2) Evaluate  $\int_0^a \int_0^{\sqrt{a^2 - x^2}} (x^2 + y^2) dy dx$  by changing into polar coordinates where  $a > 0$ . **04**
- Q.7 (a)** Trace the curve  $r = a(1 + \cos\theta)$ . **07**
- (b)** 1) Evaluate  $\int_0^4 \int_0^4 \int_0^4 (1 + xyz) dx dy dz$ . **03**
- 2) Expand  $e^x \sin y$  in powers of  $x$  and  $y$  up to second order term. **04**

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