

GUJARAT TECHNOLOGICAL UNIVERSITY
BE- SEMESTER– 1st / 2nd • EXAMINATION – SUMMER 2018

Subject Code:110014**Date: 21-05-2018****Subject Name: Calculus****Time: 02:30 pm to 05:30 pm****Total Marks: 70****Instructions:**

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** (a) (1) Evaluate $\lim_{x \rightarrow 1} \left(\frac{1}{\log x} - \frac{x}{x-1} \right)$ **03**
- (2) Show that the sequence $\left\{ \frac{n}{n^2 + 1} \right\}$ is monotonic decreasing and bounded. Is it convergent? **04**
- (b) Find expansion of $\tan \left(x + \frac{\pi}{4} \right)$ in ascending powers of x upto terms in x^4 and find approximately the value of $\tan(43^\circ)$ **07**
- Q.2** (a)(1) Find expansion of $\log(1+x)$. **03**
- (2) Test the convergence of the series $1 + \frac{2^2}{2!} + \frac{3^2}{3!} + \frac{4^2}{4!} + \dots$ **04**
- (b) Determine absolute or conditional convergence of the series. $\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n^3 + 1}$ **07**
- Q.3** (a) (1) Evaluate $\int_1^{\infty} \frac{1}{x^2} dx$ **03**
- (2) Find the linearization of $f(x, y, z) = xy + yz + xz$ at the point (1,0,0) **04**
- (b) Trace the curve $r = a(1 - \cos \theta)$, $a > 0$ **07**
- Q.4** (a)(1) Show that $f(x, y) = x^2 + 2y$ is continuous at (1,2). **03**
- (2) If $u = \tan^{-1} \left(\frac{x}{y} \right)$ where $x^2 + y^2 = a^2$ find $\frac{du}{dx}$. **04**
- (b) State Euler's theorem. If $u = \tan^{-1} \left(\frac{x^2 + y^2}{x+y} \right)$, prove that **07**
- $$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -2 \sin^3 u \cos u$$
- Q.5** (a) (1) If $u = x^2 y^3$, $x = \log t$, $y = e^t$ find $\frac{du}{dt}$ **03**
- (2) Find the equation of the tangent plane and normal line to the surface $x^2 + y^2 + z^2 = 3$ at the point (1,1,1). **04**
- (b) Change the order of integration and Evaluate for $\int_0^{4a} \int_{\frac{x^2}{4a}}^{\sqrt{ax}} xy \, dy dx$ **07**

- Q.6** (a) (1) Evaluate $\int_{-1}^1 \int_0^2 \int_0^1 (xz - y^3) dz dy dx$ **03**
- (2) State fundamental theorem of calculus. Use first fundamental theorem of calculus to find area under the curve $f(x)$ given as an integrand. **04**
- $\int_1^2 \log x dx$
- (b) Find the extreme values of the function $x^3 + 3xy^2 - 3x^2 - 3y^2 + 7$ **07**
- Q.7** (a) (1) Evaluate $\int_1^2 \int_0^1 (1 + 3xy) dx dy$ **03**
- (2) If $x = r \cos \theta, y = r \sin \theta$ find $\frac{\partial(x, y)}{\partial(r, \theta)}$ and $\frac{\partial(r, \theta)}{\partial(x, y)}$ **04**
- (b) Find the volume generated by revolving the area bounded by $2y = x^2, x = 4, y = 0$ about x -axis. **07**
