Seat No.: ________ Enrolment No.______________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- Ist /IInd SEMESTER–EXAMINATION – MAY/JUNE - 2012

Subject code: 110015 **Date: 26/05/2012**

Subject Name: Vector Calculus and Linear Algebra

Time: 10:30 am – 01:30 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Each question carry equal marks**

Q.1 (a) Solve the following homogeneous system of linear equation by using Gauss Jordan elimination **04**

$$\begin{aligned} 2x_1 + 2x_2 - x_3 \qquad + x_5 &= 0 \\ -x_1 - x_2 + 2x_3 - 3x_4 + x_5 &= 0 \\ x_1 + x_2 - 2x_3 \qquad - x_5 &= 0 \\ x_3 + x_4 + x_5 &= 0 \end{aligned}$$

(b) Attempt the following:

(i) Find A^{-1} using row operation if $A \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$ **03**

(ii) Solve the system of equation $-2b + 3c = 1$, $3a + 6b - 3c = -2$, $6a + 6b + 3c = 5$ by Gaussian elimination **03**

(c) Attempt the following:

(i) Find the rank of the matrix $A = \begin{bmatrix} 1 & -1 & 2 & 0 \\ 3 & 1 & 0 & 0 \\ -1 & 2 & 4 & 0 \end{bmatrix}$ in terms of determinates **02**

(ii) Use Cramer's rule to solve $x + 2y + z = 5$, $3x - y + z = 6$, $x + y + 4z = 7$ **02**

Q.2 (a) Define the rank and nullity. Find the rank and nullity of the matrix **05**

$$A = \begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}$$

(b) Attempt following:

(i) If $vector\ r = x\hat{i} + y\hat{j} + z\hat{k}$ then show that $\nabla r^n = n r^{n-2}(vector\ r)$ **02**

(ii) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ **03**

	(c)	<u>Attempt following:</u>	
	(i)	Show that $f_1 = 1, f_2 = e^x, f_3 = e^{2x}$, form a linearly independent set of vectors in $c^2(-\infty, \infty)$	**02**
	(ii)	Check whether the set $W = \{a_0 + a_1x + a_2x^2 + a_3x^3 \, where \;\; a_0 + a_1 + a_2 + a_3 = 0, a_i \in R\}$ is subspace of p_3	**02**
Q.3	(a)	Show that the set of all 2×2 matrices of the form $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$ with addition defined by $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} + \begin{bmatrix} c & 1 \\ 1 & d \end{bmatrix} = \begin{bmatrix} a+c & 1 \\ 1 & b+d \end{bmatrix}$ and scalar multiplication $k\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} = \begin{bmatrix} ka & 1 \\ 1 & kb \end{bmatrix}$ is a vector space.	**05**
	(b)	<u>Attempt following:</u>	
	(i)	Find a standard basis vector that can be added to the set S=$\{(1,0,3), (2,1,4)\}$ to produce a basis of R^3	**03**
	(ii)	Find the co-ordinate vector of p relative to the basis $S = \{p_1, p_2, p_3\}$ where $p = 2 - x + x^2, p_1 = 1 + x, p_2 = 1 + x^2, p_3 = x + x^2$	**02**
	(c)	<u>Attempt following:</u>	
	(i)	Find two vector in R^2 with Euclidean Norm 1 whose inner product with (-3, 1) is zero.	**02**
	(ii)	If $v_1, v_2, v_3 \ldots\ldots\ldots\ldots v_r$ are pairwise orthogonal vectors in R^n then $\lVert v_1 + v_2 + \cdots \ldots\ldots + v_r \rVert^2 = \lVert v_1 \rVert^2 + \lVert v_2 \rVert^2 + \cdots \ldots\ldots\ldots + \lVert v_r \rVert^2$	**02**
Q.4	(a)	Let R^3 have the Euclidean inner product. Use the Gram Schmidt process to transform the basis $\{u_1, u_2, u_3\}$ into an orthonormal basis $u_1 = (1,0,0), u_2 = (3,7,-2), u_3 = (0,4,1)$	**04**
	(b)	<u>Attempt following:</u>	
	(i)	Find the least squares solution of the linear system AX=B given by $x_1 - x_2 = 4$, $3x_1 + 2x_2 = 1$, $-2x_1 + 4x_2 = 3$ and find the orthogonal Projection of B on the column space of A.	**03**
	(ii)	Let the vector space P_2 have the inner product $\langle p, q \rangle = \int_{-1}^{1} p(x)q(x)dx$ (i) Find $\lVert p \rVert$ for $p = x^2$ (ii)Find $d(p,q)$ if $p = 1 \; and \; q = x$	**03**
	(c)	<u>Attempt following:</u>	
	(i)	Define the eiganvalue and eiganvector	**02**

	(ii)	Find the eigan value of A^9 for $A = \begin{bmatrix} 1 & 3 & 7 & 11 \\ 0 & 1/2 & 3 & 8 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ Find k, l and m to make A , a Hermition matrix $A = \begin{bmatrix} -1 & k & -i \\ 3-5i & 0 & m \\ l & 2+4i & 2 \end{bmatrix}$	02
Q.5	(a)	Find bases for the eiganspace of $A = \begin{bmatrix} 0 & 0 & 2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$	04
	(b)	<u>Attempt following:</u>	
	(i)	Use Cayley –Harmitlon theorem to find A^{-1} for $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$	03
	(ii)	Find a matrix P that diagonalizes $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$	03
	(c)	Find a change of variable that will reduce the quadratic form $x_1^2 - x_3^2 - 4x_1x_2 + 4x_2x_3$ to a sum of squares and express the quadratic form in terms of the new variable.	04
Q. 6	(a)	Verify Green's theorem for the field $f(x,y) = (x-y)i + xj$ and the region R bounded by the unit circle $C: r(t) = (cost)i + (sint)j, 0 \le t \le 2\pi$	04
	(b)	<u>Attempt following:</u>	
	(i)	Find the flux of $F = 4xzi - y^2j + yzk$ outward through the surface of the cube cut from the first octant by the planes $x = 1, y = 1$ and $z = 1$	03
	(ii)	Determine whether $T: R^2 \to R^2$ is a linear operator (1) $T(x,y) = (\sqrt[3]{x}, \sqrt[3]{y})$ (2) $T(x,y) = (x,0)$	03
	(c)	<u>Attempt following:</u>	
	(i)	Find the derivative of $f(x,y) = x^2 sin2y$ at the point $(1, \pi/2)$ in the direction of $v = 3i - 4j$	02
	(ii)	Use matrix multiplication to find the image of the vector (-2, 1, 2) if it is rotated -45° about the y-axis.	02
Q.7	(a)	Verify Stoke's Theorem for the hemisphere $S: x^2 + y^2 + z^2 = 9, z \ge 0$ its bounding circle $C: x^2 + y^2 = 9, z = 0$ and the field $F = yi - xj$	04
	(b)	<u>Attempt following:</u>	
	(i)	Consider the basis $S = \{v_1, v_2, v_3\}$ for R^3, $v_1 = (1,2,1), v_2 = (2,9,0)$ and $v_3 = (3,3,4)$ and let $T: R^3 \to R^2$ be the linear transform such that $T(v_1) = (1,0), T(v_2) = (-1,1), T(v_3) = (0,1)$.Find a formula for $T(x_1, x_2, x_3)$ and use that formula to find T(7,13,7).	03

(ii) Let $T: R^2 \to R^3$ be the linear transformation defined by **03**

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_2 \\ -5x_1 + 13x_2 \\ -7x_1 + 16x_2 \end{bmatrix}$$

Find the matrix for the transformation T with respect to the Basis B=$\{u_1, u_2\}$ for R^2 and $B' = \{v_1, v_2, v_3\}$ for R^3 where,

$$u_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}, v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

(c) <u>Attempt following:</u>

(i) Determine whether the linear transformation $T: R^2 \to R^2$, where $T(x,y) = (x, y, x+y)$ is one one **02**

(ii) Find the standard matrix for the linear operator on R^2,an orthogonal projection on the y-axis ,followed by a contraction with factor k=$1/3$ **02**

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