Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– 1st / 2nd • EXAMINATION – SUMMER 2013

Subject Code: 110015 **Date: 05-06-2013**

Subject Name: VECTOR CALCULUS AND LINEAR ALGEBRA

Time: 02:30 pm – 05:30 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) [02]

(1) Is the matrix $\begin{bmatrix} 1 & 0 & 4 & 6 \\ 0 & 1 & 5 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ in row echelon form or reduced row - echelon form?

(2) Find the rank of the matrix $\begin{bmatrix} 8 & 0 & 0 & 16 \\ 0 & 0 & 0 & 6 \\ 0 & 9 & 9 & 9 \end{bmatrix}$ [02]

(b) (1) Find the inverse of the matrix $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$. [02]

(2) Find the Eigen values of the matrix A^T. Is A an invertible? [02]

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 0 & 5 \\ 0 & 0 & 9 \end{bmatrix}$$

(c) (1) Express the following quadratic forms in matrix notation. [03]

(i) $x^2 - 4xy + y^2$ (ii) $2x^2 + 6xy - 5y^2$

(2) If $\phi = 3x^2y - y^3z^2$, find $grad\phi$ at the point (1,-2,-1). [03]

Q.2 (a) (1) Find the length of the arc of the curve $y = \log\sec x$ from $x = 0$ to $x = \frac{\pi}{3}$. [03]

(2) Prove that $\iint_S \vec{F}\cdot\hat{n}\,dS = 3V$,if $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$,where S is any closed surface enclosing volume V. [03]

(b) (2) If $\vec{F} = 3xy\hat{i} - y^2\hat{j}$; evaluate $\int_C \vec{F}\cdot\vec{dr}$ .Where C is the arc of the Parabola $y = 2x^2$ from (0, 0) to (1, 2). [04]

(c) Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ is both solenoidal and irrotational. [04]

Q.3 (a) Solve the following homogeneous system of equations by using Gauss-Jordan elimination. [05]

$$\begin{aligned} 2x_1 + 2x_2 - x_3 \qquad\qquad + x_5 &= 0, \\ -x_1 - x_2 + 2x_3 - 3x_4 + x_5 &= 0, \\ x_1 + x_2 - 2x_3 \qquad\qquad - x_5 &= 0, \\ x_3 + x_4 + x_5 &= 0 \end{aligned}$$

(b) Use Cramer's rule to solve:- [05]

$$\begin{aligned} x+5y \quad &= 2 \\ 11x+y+2z &= 3 \\ x+5y+2z &= 1 \end{aligned}$$

(c) What conditions must b_1, b_2, b_3 satisfy in order for the system of equations to be consistent? [04]

$$\begin{aligned} x_1+2x_2+3x_3 &= b_1 \\ 2x_1+5x_2+3x_3 &= b_2 \\ x_1 \qquad\quad +8x_3 &= b_3 \end{aligned}$$

Q.4 (a) *Show that the set of all* 2×2 *matrices of the form* $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$ with addition defined by $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$ + $\begin{bmatrix} c & 1 \\ 1 & d \end{bmatrix}$ = $\begin{bmatrix} a+c & 1 \\ 1 & b+d \end{bmatrix}$ and scalar multiplication defined by $k\cdot\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} = \begin{bmatrix} ka & 1 \\ 1 & kb \end{bmatrix}$ is a vector space. [06]

(b) *Determine which of the following are subspaces of* R^3. [04]

(1) all the vector of the form (a, b, c), where b = a+c.

(2) all the vector of the form (a, b, c), where b = a+c+1.

(c) Let $v_1 = 1-3x+2x^2$, $v_2 = 1-x+4x^2$, $v_3 = 1-7x$ .Show that the set $S=\{v_1, v_2, v_3\}$ is a basis for p_2 . [04]

Q.5 (a) Use the inner product $\langle f, g\rangle = \int_0^1 f(x)g(x)dx$; to compute $d(f,g)$ *and* $\|f\|$. [04]

Where $f(x)=x$, $g(x)=e^x$ *in* $C[0,1]$.

(b) Let R^3 have the Euclidean inner product. Use the Gram-Schmidt process to transform the basis $\{u_1, u_2, u_3\}$ into an orthonormal basis. [05]

Where $u_1=(1,1,1), u_2=(-1,1,0), u_3=(1,2,1)$.

(c) [05]

Verify that the vectors $u_1=(1,-1,2,-1), u_2=(-2,2,3,2), u_3=(1,2,0,-1), u_4=(1,0,0,1)$ form an Orthogonal basis for R^4 with the Euclidean inner product. Also convert it to an orthonormal set by normalizing the vectors.

Q. 6 (a) Find the eigen values and bases for the eigenspaces of the matrix $\begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$. [05]

(b) Find matrix P that diagonalizes $A=\begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$. Also determine $P^{-1}AP$. [05]

(c) [04]

Using Cayley-Hamilton theorem, find A^{-1}; $A=\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

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Q.7 (a) Let $T_A : R^6 \rightarrow R^4$ be multiplication by $A = \begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}$ [05]

Find the rank and nullity of T_A.

(b) Consider the basis $S = \{v_1, v_2\}$ for R^2, where $v_1 = (-2,1)$ *and* $v_2 = (1,3)$ .Let $T : R^2 \rightarrow R^3$, be the linear operator such that $T(v_1) = (-1,2,0)$ *and* $T(v_2) = (0,-3,5)$ find the formula T (x_1 , x_2),and use that formula to find T (2, -3). [05]

(c) Show that the following functions are linear transformations. [04]

(i) $T : R^2 \rightarrow R^2$, where $T(x, y) = (x + 3y, 3x - y)$

(ii) $T : R^2 \rightarrow R^2$, where $T(x, y) = (2x - y,\ x - y)$

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