Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER– 2nd • EXAMINATION – SUMMER 2018

Subject Code:110015 **Date: 17-05-2018**

Subject Name: Vector Calculus and Linear Algebra

Time: 02:30 pm to 05:30 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q-1

(a) **1** If $A = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 2 & 0 \\ 12 & 15 & 3 \end{bmatrix}$ then find the eigen values of A and hence find eigen values of A^5 and A^{-1}. **4**

2 Prove that $A = \begin{bmatrix} 1 & 3+4i & -2i \\ 3-4i & 2 & 9-7i \\ 2i & 9+7i & 3 \end{bmatrix}$ is a Hermitian matrix. **3**

(b) **1** Solve the following system of equations **4**

$$x + 5y = 2$$
$$11x + y + 2z = 3$$
$$x + 5y + 2z = 1$$

Using Gauss elimination method

2 Determine whether or not vectors $(1, -2, 1), (2, 1, -1), (7, -4, 1)$ in R^3 are linearly independent. **3**

Q-2

(a) **1** Investigate for what values of λ and μ the equations **4**

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

have (i) a unique solution (ii) no solution

2 Obtain the reduced row echelon form of the matrix $A = \begin{bmatrix} 1 & 3 & 2 & 2 \\ 1 & 2 & 1 & 3 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 8 \end{bmatrix}$ **3**

(b) Prove that the set of all 2x2 matrices of the form $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$ with the operations defined as **7**

$\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} + \begin{bmatrix} c & 1 \\ 1 & d \end{bmatrix} = \begin{bmatrix} a+b & 1 \\ 1 & b+d \end{bmatrix}$ & k $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} = \begin{bmatrix} ka & 1 \\ 1 & kb \end{bmatrix}$ is a vector space.

Q-3

(a) For the basis $B = \{v_1, v_2, v_3\}$ of R^3 where $v_1 = (1,1,1)$, $v_2 = (1,1,0)$ and $v_3 = (1,0,0)$. Let $T: R^3 \rightarrow R^3$ be a linear transformation such that $(v_1) = (2,-1,4)$, $T(v_2) = (3,0,1)$, $T(v_3) = (-1,5,1)$. Find a formula for $T(x_1, x_2, x_3)$ and use it to find $T(2,4,-1)$. **7**

(b) **1** (i) Find the Euclidean inner product u.v where **4**

$u = (3,1,4,\ -5),\quad v = (2,\ 2,-4,-3)$

(ii) For which values of k are $u = (2,1,3)$ and $v = (1,7,k)$ orthogonal?

2 For $u = (2,1,3), v = (2,-1,3)$ verify Cauchy-Schwarz inequality holds. **3**

Q-4

(a) **1** Find eigen values and eigenvectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ **4**

2 Define Symmetric matrix and Skew-symmetric matrix by giving example. **3**

(b) **1** Translate and rotate the coordinate axes, if necessary, to put the conic $9x^2 - 4xy + 6y^2 - 10x - 20y = 5$ in standard position. Find the equation of the conic in the final coordinate system. **4**

2 Use Cayley-Hamilton theorem to find A^{-1} fro $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ **3**

Q-5

(a) **1** Using Gram-Schmidt process, construct an orthonormal basis for R^3, whose basis is the set $\{(1,1,1),(1,-2,1),(1,2,3)\}$ **4**

2 Show that $w = (9,2,7)$ is a linear combination of the vectors $u = (1,2,-1)$ and $v = (6,4,2)$ in R. **3**

(b) **1** Find the least squares solution of the linear system $Ax = b$, and find the orthogonal projection of b onto the column space of A. **4**

$$A = \begin{bmatrix} 2 & -2 \\ 1 & 1 \\ 3 & 1 \end{bmatrix}, b = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

2 Which of the following are subspaces of R^3? **3**

(i) All the vectors of the form (a,b,c) where $b = a + c$

(ii) All the vectors of the form (a,b,c) where $b = a + c + 1$

Q-6

(a) Verify Green's theorem for the function $F = (x^2 + y^2)\hat{\imath} - 2xy\hat{\jmath}$, where C is the rectangle in the xy-plane bounded by $y = 0, y = b, x = 0$ and $x = a$. **7**

(b) **1** Verify Stoke's theorem for $F = (x^2 - y^2)\,\hat{\imath} + 2xy\hat{\jmath}$ in the rectangular region $x = 0, y = 0, x = a, y = b$. **4**

2 Find A^{-1} using Gauss-Jordan method if $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ **3**

Q-7

(a) **1** (i) Find $grad(\emptyset) = \log(x^2 + y^2 + z^2)$ at the point $(1,0,-2)$ **4**

(ii) Find a unit vector normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point $(1,2,-1)$

2 Find the directional derivative of the divergence of $\bar{F}(x,y,z) = xy\hat{\imath} + xy^2\,\hat{\jmath} + z^2\hat{k}$ at the point $(2,1,2)$ in the direction of the outer normal to the sphere $x^2 + y^2 + z^2 = 9$. **3**

(b) **1** Show that $\bar{F} = (y^2 - z^2 + 3yz - 2x)\hat{\imath} + (3xz + 2xy)\hat{\jmath} + (3xy - 2xz + 2z)\hat{k}$ is both solenoidal and irrotational. **4**

2 Evaluate $\iint_S \bar{F} \,.\, d\bar{s}$, where $\bar{F} = (2x + 3z)\hat{\imath} - (xz + y)\hat{\jmath} + (y^2 + 2z)\hat{k}$ and S is the surface of the sphere having center at $(3, -1, 2)$ and radius 3. **3**

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