Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY
BE- SEMESTER– 1st / 2nd (NEW) • EXAMINATION – SUMMER 2016

Subject Code: 2110014 Date:02/06/2016
Subject Name: Calculus
Time: 02:30 PM to 05:30 PM Total Marks: 70

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Sixquestions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **Objective Question (MCQ)**

(a) Choose the most appropriate answer out of the following options given for each part of the question: **07**

1. The sum of the series $1-\frac{1}{2}+\frac{1}{2^2}-\frac{1}{2^3}+\cdots$ is _______

(A) $2/3$ (B) $3/2$ (C) $1/2$ (D) None of these

2. The series $x+\frac{x^3}{3!}+\frac{x^5}{5!}+\cdots$ represent expansion of _______

(A) $\sin x$ (B) $\cos x$ (C) $\sinh x$ (D) $\cosh x$

3. The value of the $\lim_{x\to\infty}\frac{\sin x}{x}$ is _______

(A) 1 (B) -1 (C) 0 (D) None of these

4. The curve $r=\cos 2\theta$ has _________ petals.

(A) 2 (B) 4 (C) 3 (D) None of these.

5. If $x=r\cos\theta,\ \ y=r\sin\theta$, then $\frac{\partial(r,\theta)}{\partial(x,y)}=$ __________

(A) r (B) $1/r$ (C) 0 (D) None of these

6. If $u=x^3\cos\left(y/x\right)$, then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$ __________

(A) u (B) $2u$ (C) $3u$ (D) None of these

7. The equation of a cylindrical surface $x^2+y^2=9$ becomes ______ when converted to cylindrical polar coordinates.

(A) $r=9$ (B) $r^2=9$ (C) $r=\pm 3$ (D) $r=3$

(b) Choose the most appropriate answer for the following questions: **07**

1. The series $\sum_{n=1}^{\infty}\frac{1}{(\log n)^n}$ is _________

(A) oscillatory (B) divergent (C) convergent (D) None of these

2. If in the equation of a curve, x occurs only as an even power then the curve is symmetrical about

(A) x-axis (B) y – axis (C) Origin (D) None of these

3. The value of the $\lim_{x\to 0}(\cos x)^{\cot x}$ is _______

(A) 0 (B) 1 (C) ∞ (D) None of these

4. The integral $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$ if ________

(A) f is an odd function (B) f is neither even nor odd function
(C) f is an even function (D) None of these

5. The volume of the solid generated by revolving the region between the y – axis and the curve $x = 2\sqrt{y},\ 0 \le y \le 4,$ about the y – axis.

(A) 2π (B) 32π (C) 16π (D) None of these

6. $\int_0^2 \int_0^{x^2} e^{y/x}\, dy\, dx$ is equal to ____________

(A) $e^2 - 1$ (B) e^2 (C) $e^2 + 1$ (D) e^{-2}

7. A point (a, b) is said to be a saddle point if at (a, b)

(A) $rt - s^2 > 0$ (B) $rt - s^2 = 0$ (C) $rt - s^2 < 0$ (D) $rt - s^2 \ge 0$

Q.2 **(a)** Expand $\tan\left(\frac{\pi}{4} + x\right)$ in powers of x by using the Taylor's series. Also, find the value of $\tan 44°$. **04**

(b) Evaluate: $\lim_{x \to 0} \left(\frac{1}{x}\right)^{\tan x}$. **03**

(c) (1) Test for convergence the series $x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots,\quad x > 0.$ **03**

(2) Trace the curve $r^2 = a^2 \sin 2\theta$. **04**

Q.3 **(a)** If $\theta = t^n e^{-\frac{r^2}{4t}}$ then find n so that $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \theta}{\partial r}\right) = \frac{\partial \theta}{\partial t}$. **04**

(b) Determine the continuity of the function **03**

$$f(x, y) = \begin{cases} (x^2 + y^2)\sin\left(\frac{1}{x^2 + y^2}\right), & (x, y) \ne (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

at origin.

(c) (1) If $u = \tan^{-1}(x^2 + 2y^2)$, show that $x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy} = 2\sin u . \cos 3u$ **04**

(2) If $u = f\left(\frac{y - x}{xy}, \frac{z - x}{xz}\right)$, show that $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0.$ **03**

Q.4 **(a)** Find the extreme values of the function $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x.$ **04**

(b) Find the equation of the tangent plane and the normal line of the surface $\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$ at $(-2, 1, -3)$. **03**

(c) (1) Show that the rectangular solid of maximum volume that can be inscribed in a sphere is cube. **04**

(2) Expand $xy^2 + xy + 3$ in powers of $(x - 1)$ and $(y + 2)$ using Taylor's series expansion. **03**

Q.5 **(a)** Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n + 1}}$ **04**

(b) Test the convergence of the series $\sum_{n=1}^{\infty} \sqrt{n^4 + 1} - \sqrt{n^4 - 1}$. **03**

(c) (1) Define geometric series and show that the geometric series is : **04**

(i) convergent if $|r| < 1$, (ii) divergent if $r \ge 1$

(2) Express $(x - 1)^4 + 2(x - 1)^3 + 5(x - 1) + 2$ in ascending power of x. **03**

Q.6 **(a)** Evaluate $\iint_R (x+y)\,dy\,dx,$ where R is the region bounded by $x=0,\ \ x=2,\ \ y=x,\ \ y=x+2.$ **04**

(b) Evaluate $\iint_R r^3 \sin 2\theta \, dr\ d\theta$ over the area bounded in the first quadrant between the circles $r=2$ and $r=4.$ **03**

(c) (1) Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z\ dz\ dx\ dy$ **04**

(2) Evaluate $\lim_{x \to \frac{\pi}{2}} (\cos x)^{\frac{\pi}{2}-x}.$ **03**

Q.7 **(a)** Find the volume of the solid generated by revolving the curve $x^{2/3}+y^{2/3}=a^{2/3}$ about the $x-$axis. **04**

(b) Find the area lying inside the circle $r=a\sin\theta$ and outside the cardioid $r=a(1-\cos\ \theta).$ **03**

(c) (1) Evaluate $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}}\,dy\,dx$ by changing the order of integration. **04**

(2) Check the convergence of $\int_0^5 \frac{1}{x^2}\,dx.$ **03**

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