

GUJARAT TECHNOLOGICAL UNIVERSITY
BE- SEMESTER 1st / 2nd EXAMINATION (NEW SYLLABUS) – SUMMER - 2017

Subject Code: 2110014**Date: 01/06/2017****Subject Name: Calculus****Time: 2:30 PM to 05:30 PM****Total Marks: 70****Instructions:**

1. Question No. 1 is compulsory. Attempt any four out of remaining Six questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1	Objective Question (MCQ)	Mark
	(a) Choose the appropriate answer for the following question.	07
	1. Infinite series $1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots$ is _____.	
	(A) Divergent (B) Convergent (C) Oscillation (D) None of these	
	2. The series $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$ represent expansion of _____.	
	(A) e^x (B) $\log(1+x)$ (C) $\sin x$ (D) $\cos x$	
	3. The value of the $\lim_{x \rightarrow 0} \left(\frac{\sin 2x}{x} \right) =$ _____.	
	(A) 2 (B) 1 (C) -1 (D) $\frac{1}{2}$	
	4. Asymptote parallel to y-axis of the curve $y = \frac{3x^2}{x-2}$ is the line _____.	
	(A) $x = 0$ (B) $y = 0$ (C) $x = 2$ (D) $y = 2$	
	5. $f(x) = x $ is _____ at origin.	
	(A) continuous (B) discontinuous (C) differentiable (D) None of these	
	6. Curve $y^2(a+x) = x^2(b-x)$ is symmetric about _____.	
	(A) x - axis (B) y - axis (C) line $x = b$ (D) line $x = -a$	
	7. The curve increases strictly in the interval in which $\frac{dy}{dx}$ _____.	
	(A) < 0 (B) > 0 (C) $= 0$ (D) None of these	
	(b) Choose the appropriate answer for the following question.	07
	1. The values of $\int_0^\infty e^{-x} \cos(2x) dx =$ _____.	
	(A) 0 (B) $-\frac{1}{5}$ (C) $\frac{1}{5}$ (D) $\frac{2}{5}$	
	2. What does the region $\int_1^2 \int_1^2 dx dy =$ _____.	
	(A) rectangle (B) square (C) circle (D) triangle	
	3. The values of the $\lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\cos x}{x - \frac{\pi}{2}} \right) =$ _____.	
	(A) 0 (B) 1 (C) -1 (D) $\frac{\pi}{2}$	

4. The function $f(x,y) = x^2y f(y/x)$ is homogeneous of degree is ____ .
 (A) 0 (B) 1 (C) 2 (D) 3
5. The equation of the form $f(xy) = c$, then $\frac{dy}{dx} =$ ____ .
 (A) $-\frac{f_x}{f_y}$ (B) $\frac{f_x}{f_y}$ (C) $-\frac{f_y}{f_x}$ (D) $\frac{f_y}{f_x}$
6. The values of $\int_0^1 \int_0^1 (3x^2 - 2y^2) dx dy$ is ____ .
 (A) 0 (B) 1 (C) -1 (D) $\frac{1}{3}$
7. If $x = u + 3v$, $y = v - u$ then the values of $\frac{\partial(x,y)}{\partial(u,v)}$ is ____ .
 (A) 4 (B) -1 (C) 5 (D) 7

Q.2 (a) Expand $\log(\sec x)$ in power of x . 04

(b) Evaluate $\lim_{x \rightarrow 0} \left(\frac{1^x + 2^x + 3^x}{3} \right)^{\frac{1}{x}}$. 03

(c) (i) Trace the curve $y^2(2a - x) = x^3$. 04

(ii) determine $\int_0^3 \frac{1}{\sqrt{3-x}} dx$ converge or diverges. 03

Q.3 (a) If $f(x,y) = x^2y + xy^2$ then find $f_x(1,2)$ and $f_y(1,2)$ by definition. 04

(b) Check the continuity for the following function at $(0,0)$ 03

$$f(x,y) = \frac{2xy}{x^2 + y^2}, \quad (x,y) \neq (0,0)$$

$$= 0, \quad (x,y) = (0,0)$$

(c) (i) if $u = \sin^{-1} \left(\frac{\frac{1}{x^4} + \frac{1}{y^4}}{\frac{1}{x^5} + \frac{1}{y^5}} \right)$ then find the values of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ and 04

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}.$$

(ii) if $u = \frac{e^{x+y+z}}{e^x + e^y + e^z}$ then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 2u$. 03

Q.4 (a) Find the extreme values of $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$. 04

(b) Find the equation of the tangent plane and normal line to the surface $2x^2 + y^2 + 2z = 3$ at $(2, 1, -3)$ 03

(c) (i) Find a point on the plane $2x + 3y - z = 5$ which is nearest to the origin. 04

(ii) Expanded e^{xy} in power of $x - 1$ and $y - 1$ using Taylor's expansion. 03

- Q.5** (a) Test for the convergence the series; $\frac{1}{1.2.3} + \frac{x}{4.5.6} + \frac{x^2}{7.8.9} + \dots$ **04**
- (b) Test for the convergence the series; **03**
- $$\sum_{n=1}^{\infty} [\sqrt[3]{n^3 + 1} - n]$$
- (c) (i) Determine absolute or conditional convergence of the series; **04**
- $$\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n^3 + 1}$$
- (ii) Test the convergence for $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \dots$ **03**
- Q.6** (a) Evaluate $\iint r \sqrt{a^2 - r^2} \, dr d\theta$ over the upper half of the circle $r = a \cos \theta$. **04**
- (b) Sketch the region of integration and evaluate **03**
- $$\int_1^4 \int_0^{\sqrt{x}} \frac{3}{2} e^{\frac{y}{\sqrt{x}}} \, dy dx .$$
- (c) (i) Evaluate the integral $\int_0^2 \int_{\frac{y}{2}}^1 e^{x^2} \, dx dy$ by changing the order of **04**
- integration.
- (ii) Evaluate the integral $\int_0^2 \int_1^z \int_0^{yz} xyz \, dx dy dz$. **03**
- Q.7** (a) Find the area included between the curve $y^2(2a - x) = x^3$ and its asymptote. **04**
- (b) Find the volume of a solid generated by revolving the cardioid **03**
- $$r = a(1 + \cos \theta)$$
- about the initial line.
- (c) Use triple integration to find the volume of the solid within the cylinder **07**
- $$x^2 + y^2 = 9$$
- between the planes $z = 1$ and $x + z = 1$.