GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER–I &II (NEW) EXAMINATION – SUMMER-2019
Subject Code: 2110014
Subject Name: Calculus
Time: 10:30 AM TO 01:30 PM
Total Marks: 70
Instructions:
1. Question No.1 is compulsory. Attempt any four out of remaining six questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 Objective Question (MCQ)

(a) 07

1. For the Jacobian $J$, value of the $J \cdot J'$ is
   (a) 1 (b) -1 (c) 0 (d) 2

2. Value of $\frac{dy}{dx}$ for $ax^2 + 2hxy + by^2 = 1$ is
   (a) $\frac{ax + hy}{hx + by}$ (b) $\frac{ax + hy}{hx + by}$ (c) $\frac{ax + hy}{hx + by}$ (d) $\frac{hx + by}{ax + hy}$

3. $u = \sin^{-1} \frac{x}{y}$ is a homogeneous function of degree
   (a) $\frac{1}{2}$ (b) 0 (c) 1 (d) -1

4. The curve $r = 2$ is
   (a) straight line (b) point at distance ‘2’ on initial line (c) circle with centre origin and radius 2 (d) cardioid

5. If $x = rcos\theta$, $y = rsin\theta$, then which is correct?
   (a) $r = x^2 + y^2$, $\theta = x$ (b) $r = \sqrt{x^2 + y^2}$, $\theta = tan^{-1} \frac{y}{x}$
   (c) $r = x^2 + y^2$, $\theta = tan^{-1} \frac{x}{y}$ (d) $r = \sqrt{x^2 + y^2}$, $\theta = tan^{-1} \frac{y}{x}$

6. Infinite Sequence $\{1,1,1,\ldots\}$ is
   (a) convergent (b) divergent (c) oscillatory (d) None of these

7. Infinite Series $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \ldots$ is
   (a) convergent (b) divergent (c) oscillatory (d) None of these

(b) 07

1. Infinite series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots$ is
   (a) convergent (b) divergent (c) oscillatory (d) None of these

2. Curve $(y - 1)^2 = x - 5$ is symmetric to
   (a) X-axis (b) Y-axis (c) line $y = -x$ (d) Y-axis

3. $\lim_{x \to 0} \frac{\tan \pi x}{x}$
   (a) $\frac{1}{\pi}$ (b) 0 (c) $\infty$ (d) $\pi$

4. The sum of the series $\sum_{n=0}^{\infty} \frac{1}{2^n}$ is
   (a) $\infty$ (b) $\frac{1}{2}$ (c) 2 (d) 1

5. The Maclaurin series for the function $(x + 1)^2$ is
   (a) $1 + x + x^2$ (b) $1 + 2x + x^2$ (c) $1 + x$ (d) $x + x^2$

6. The straight line $y = 2$ is revolved about x-axis between $0 \ll x \ll 4$. The generated solid is
   (a) cone (b) sphere (c) cuboid (d) cylinder

7. For a series $\sum_{n=1}^{\infty} a_n$, if $\lim_{n \to \infty} a_n \neq 0$, then
   (a) series is convergent (b) series is divergent (c) sum of series is finite number (d) series is conditionally convergent
Q.2 (a) Find the Taylor series for \( f(x) = \frac{1}{x} \) at \( a = 2 \).
(b) Is the series absolutely convergent or conditionally convergent?
\[
1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + - \cdots
\]
(c) (i) Discuss the convergence of the series
\[
\frac{x}{1 \cdot 2} + \frac{x^2}{2 \cdot 3} + \frac{x^3}{3 \cdot 4} + \cdots
\]
(ii) Find the Radius of convergence for the series \( \sum_{n=1}^{\infty} \frac{x^n}{n!} \).

Q.3 (a) Evaluate \( \lim_{x \to 0} x \log x \)
(b) Trace the curve \( y^2(x + a) = x^2(a - x), \ a > 0 \).
(c) Prove that the series \( \sum_{n=1}^{\infty} \frac{1}{n^p} \) is convergent if \( p > 1 \) and divergent if \( p < 1 \).

Q.4 (a) Evaluate \( \int_{0}^{3} \frac{dx}{(x-1)^{3/2}} \).
(b) Find the equation of the tangent plane and normal line to the surface \( x^2 + y^2 + z - 9 = 0 \) at \( (1, 2, 4) \).
(c) (i) Evaluate \( \int_{-\infty}^{\infty} \frac{dx}{1 + x^2} \).
(ii) Evaluate \( \lim_{x \to \frac{\pi}{2}} (1 - \cos x) \tan x \)

Q.5 (a) If \( u = f(x - y, y - z, z - x) \), prove that \( u_x + u_y + u_z = 0 \).
(b) Find maximum and minimum values.
\( f(x, y) = 2(x^2 - y^2) - x^4 + y^4 \)
(c) If \( u = \tan^{-1}\left(\frac{x^2 + y^2}{x - y}\right) \), prove that
(i) \( xu_x + yu_y = \sin 2u \)
(ii) \( x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = 2 \sin u \cos 3u \)

Q.6 (a) The region between the curve \( y = \sqrt{x}, 0 \ll x \ll 4 \) and the x-axis is revolved about the x-axis to generate a solid. Find its volume.
(b) Using volume by slicing method, find the volume of a cylinder with radius ‘r’ and height ‘h’.
(c) Evaluate \( \int_{R} x \, dxdy \); \( R \) is triangle \( (0,0),(1,0),(1,1) \) using transformations \( x = u, y = uv \).

Q.7 (a) Evaluate \( \int_{r}^{3} \int d\theta \) over the area bounded between the circles \( r = 2\cos \theta \) and \( r = 4\cos \theta \).
(b) Evaluate \( \int_{0}^{1} \int_{0}^{1-x} (x+y)^2 \, dz \, dy \, dx \)
(c) Change the order of integration and evaluate.
\( \int_{0}^{1} \int_{0}^{2-x} xy \, dy \, dx \)
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