Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE SEMESTER– 1st/2nd (NEW SYLLABUS) EXAMINATION – SUMMER 2015

Subject Code: 2110015 **Date:15/06/2015**

Subject Name: Vector Calculus and Linear Algebra

Time:10.30am-01.00pm **Total Marks: 70**

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Answer the following MCQ **07**

1. The angle between $\bar{u} = (-1,1,2,-2)$ and $\bar{v} = (2,-1,-1,3)$ is_____

(a) 3 rad (c) 2.68 rad
(b) 3.68 rad (d) 2 rad

2. Which of the following matrix is orthogonal?

(a) $\begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$ (c) $\begin{bmatrix} 1/9 & \sqrt{8}/9 \\ \sqrt{5}/9 & 2/9 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$

3. If a square matrix A is involutory then $A^2 =$ _______

(a) A (c) A^T
(b) I (d) A^{-1}

4. A homogeneous system of equations have at least _____solutions

(a) 1 (c) 3
(b) 2 (d) 4

5. The rank of $\begin{bmatrix} 1 & 3 & 6 \\ 2 & 1 & 6 \end{bmatrix}$ is

(a) 1 (c) 3
(b) 2 (d) No rank

6. If 3 is the eigen value of A then the eigen value of $A + 3I$ is

(a) 9 (c) 0
(b) 6 (d) 27

7. Which of the following is a subspace of R^2 under standard operations

(a) R^3 (c) M_{22}
(b) P_2 (d) R

(b) Attempt the following MCQ **07**

1. If R is a vector space then which of the following is a trivial subspace of R?

(a) $\{\bar{0}\}$ (c) $\{\bar{0},\bar{1}\}$
(b) R (d) $\{\bar{1}\}$

2. The set $S = \{1, x, x^2\}$ spans which of the following?

(a) R^2 (c) P_2
(b) M_{22} (d) R

3. The dimension of the solution space of $x - 3y = 0$ is__________

(a) 1 (c) 3
(b) 2 (d) 4

4. The mapping $T : R^3 \to R^3$ defined by $T(v_1, v_2, v_3) = (v_1, v_2, 0)$ is called as

(a) Reflection (c) Rotation
(b) Magnification (d) Projection

5. If $\langle \bar{u}, \bar{v} \rangle = 9u_1v_1 + 4u_2v_2$ is the inner product on R^2 then it is generated by

(a) $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 3 \\ 2 & 0 \end{bmatrix}$

6. The divergence of $\bar{F} = xyzi + 3x^2 yj + (xz^2 - y^2 z)k$ at (2,-1,1) is

(a) $yz + 3x + 2xz$ (c) $yz + 3x^2 + (2xz - y^2)$

(b) $yz + xy$ (d) $xy - yz$

7. If $\bar{F}$ is conservative field then $curl\bar{F} =$ __________

(a) i (c) k

(b) j (d) $\bar{0}$

Q.2 **(a)** Is $A = \begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$ orthogonal? If not, can it be converted into an orthogonal matrix? **03**

(b) Solve the following system: $x + y + z = 3, x + 2y - z = 4, x + 3y + 2z = 4.$ **04**

(c)(i) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ -3 & 1 & 5 & 2 \\ -2 & 3 & 9 & 2 \end{bmatrix}$ **03**

(ii) Find the inverse of $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ using row operations. **04**

Q.3 **(a)** Does $W = \{(x, y, z) / x^2 + y^2 + z^2 = 1\}$ a subspace of R^3 with the standard operations? **03**

(b) Find the projection of $\bar{u} = (1,-2,3)$ along $\bar{v} = (2,5,4)$ in R^3. **04**

(c) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ **07**

Q.4 **(a)** Find the least square solution for the system $4x_1 - 3x_2 = 12, 2x_1 + 5x_2 = 32, 3x_1 + x_2 = 21.$ **03**

(b) Determine the dimension and basis for the solution space of the system $x_1 + 2x_2 + x_3 + 3x_4 = 0, 2x_1 + 5x_2 + 2x_3 + x_4 = 0, x_1 + 3x_2 + x_3 - x_4 = 0.$ **04**

(c) Check whether $V = R^2$ is a vector space with respect to the operations $(u_1, u_2) + (v_1, v_2) = (u_1 + v_1 - 2, u_2 + v_2 - 3)$ and $\alpha(u_1, u_2) = (\alpha u_1 + 2\alpha - 2, \alpha u_2 - 3\alpha + 3), \alpha \in R$. **07**

Q.5 **(a)** Consider the inner product space P_2. Let $\overline{p_1} = a_2x^2 + a_1x + a_0$ and $\overline{p_2} = b_2x^2 + b_1x + b_0$ are in P_2, where $\langle \overline{p_1}, \overline{p_2} \rangle = a_2b_2 + a_1b_1 + a_0b_0$. Find the angle between $2x^2 - 3$ and $3x + 5$. **03**

(b) Find the rank and nullity of the matrix $A=\begin{bmatrix}2 & 0 & -1\\ 4 & 0 & -2\\ 0 & 0 & 0\end{bmatrix}$ and verify the dimension theorem. **04**

(c) Let $T: R^2 \to R^3$ be the linear transformation defined by $T(x_1, x_2) = (x_2, -5x_1 + 13x_2, -7x_1 + 16x_2)$. Find the matrix for the transformation T with respect to the bases $B = \{(3,1),(5,2)\}$ for R^2 and $B' = \{(1,0,-1),(-1,2,2),(0,1,2)\}$ for R^3. **07**

Q.6 (a) Find the arc length of the portion of the circular helix $\bar{r}(t) = \cos t i + \sin t j + tk$ from *t=0 to t=π.* **03**

(b) A vector field is given by $\bar{F} = (x^2 + xy^2)i + (y^2 + x^2 y)j$. Show that $\bar{F}$ is irrotational and find its scalar potential. **04**

(c)(i) Let R^3 have the Euclidean inner product. Transform the basis {(1,1,1),(1,-2,1),(1,2,3)} into an orthogonal basis using Gram-Schmidt process. **05**

(ii) Express the following quadratic form in matrix notation: **02**

$2x^2 + 5y^2 - 6z^2 - 2xy - yz + 8zx.$

Q.7 (a) If $\phi = xyz - 2y^2 z + x^2 z^2$, find $div(grad\phi)$ at the point (2,4,1). **03**

(b) Use Green's theorem to evaluate $\oint_C [x^2 y dx + y^3 dy]$, where C is the closed path formed by *y=x* and $y = x^3$ from (0,0) to (1,1). **04**

(c) Verify Stoke's theorem for $\bar{F} = (y - z + 2)i + (yz + 4)j - xzk$ over the surface of the cube *x=0, y=0, z=0, x=2, y=2, z=2* above the *xy* - plane.(that is open at bottom) **07**

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