GUJARAT TECHNOLOGICAL UNIVERSITY
BE- SEMESTER 1st / 2nd EXAMINATION (NEW SYLLABUS) – SUMMER - 2017

Subject Code: 2110015
Subject Name: Vector Calculus & Linear Algebra
Time: 2:30 PM to 05:30 PM

Instructions:
1. Question No. 1 is compulsory. Attempt any four out of remaining Six questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 Objective Question (MCQ)

(a) 07
1. Let A be a non singular matrix of order $n \times n$ then $|adj A|$ is equal to
   (a) 0  (b) 1  (c) 2  (d) $|A|^{n-1}$
2. Let A be a skew symmetric matrix of odd order then $|A|$ is equal to
   (a) 0  (b) 1  (c) 2  (d) -1
3. The maximum possible rank of a singular matrix of order 3 is
   (a) 0  (b) 1  (c) 2  (d) 3
4. Let A be a square matrix of order n with rank r where r < n, then the number of independent solutions of the homogeneous system of equation AX = 0 is
   (a) n  (b) r  (c) $n-r$  (d) 1
5. The dimension of the polynomial space $P_3$ is
   (a) 1  (b) 2  (c) 3  (d) 4
6. Let T: $R^2 \rightarrow R^2$ be a linear transformation defined by $T(x, y) = (3x, 3y)$ then T is classified as
   (a) Reflection  (b) Magnification  (c) Rotation  (d) Projection
7. Let T: $R^3 \rightarrow R^3$ be a linear transformation defined by $T(x, y, z) = (y, -x, 0)$ then the dimension of $R(T)$ is
   (a) 0  (b) 1  (c) 2  (d) 3

(b) 07
1. The product of the eigen values of $\begin{bmatrix} 1 & 4 & 3 \\ 0 & 2 & 5 \\ 0 & 0 & 2 \end{bmatrix}$ is
   (a) 1  (b) 2  (c) 3  (d) 4
2. Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ then the eigen values of $A + 3I$ are
   (a) 1, 3  (b) 2, 4  (c) 4, 6  (d) 2, 3
3. Let T: $R^2 \rightarrow R^2$ be a linear transformation defined by $T(x, y) = (y, x)$ then it is
   (a) One to one  (b) Onto  (c) Both  (d) Neither
4. For vectors $u$ and $v$, $\|u + v\|^2 - \|u - v\|^2$ is
   (a) $<u, v>$  (b) $2 <u, v>$  (c) $3 <u, v>$  (d) $4 <u, v>$
5. If $\|u + v\|^2 = \|u\|^2 + \|v\|^2$, then the vectors $u$ and $v$ are
   (a) Parallel  (b) Orthogonal  (c) dependent  (d) Co linear
6. The magnitude of the maximum directional derivative of the function $2x + y + 2z$ at the point $(1, 0, 0)$ is
   (a) 0  (b) 1  (c) 2  (d) 3
7. For vector point function $\vec{F}$, divergence of $\vec{F}$ is obtained by
   (a) $\nabla \cdot \vec{F}$  (b) $\nabla \times \vec{F}$  (c) $\nabla \vec{F}$  (d) $\nabla^2 F$

Q.2 (a) Express the matrix $\begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & 7 \end{bmatrix}$ as a sum of a symmetric and a skew–symmetric matrix.
Find the inverse of \[
\begin{bmatrix}
2 & 1 & 3 \\
3 & 1 & 2 \\
1 & 2 & 3
\end{bmatrix}
\] using Gauss Jordan method. Determine the values of \( k \), for which the equations
\[3x - y + 2z = 1, - 4x + 2y - 3z = k, \quad \text{and} \quad 2x + z = k^2\]
possesses solution. Find the solutions in each case.

Q.3  (a) Show \((9, 2, 7)\) as a linear combination of \((1, 2, -1)\) and \((6, 4, 2)\).
(b) Find a basis for the null space of
\[
\begin{bmatrix}
2 & -1 & -2 \\
-4 & 2 & 4 \\
-8 & 4 & 8
\end{bmatrix}
\]
(c) Check whether the set of all ordered pairs of real numbers \((x, y)\) with the operations defined as \((x_1 + y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)\) and \(k(x, y) = (2kx, 2ky)\) is a vector space. If not, list all the axioms which are not satisfied.

Q.4  (a) Check whether the vectors \(1, \sin^2 x \) and \(\cos 2x\) are linearly dependent or independent.
(b) Find the eigen values and eigen vectors of
\[
\begin{bmatrix}
1 & 2 & 2 \\
0 & 2 & 1 \\
-1 & 2 & 2
\end{bmatrix}
\]
(c) Verify Cayley Hamilton theorem for
\[
\begin{bmatrix}
6 & -1 & 1 \\
-2 & 5 & -1 \\
2 & 1 & 7
\end{bmatrix}
\]
and hence find \(A^{-1}\) and \(A^4\).

Q.5  (a) Verify parallelogram law for \(\begin{bmatrix}
-1 & 2 \\
6 & 1
\end{bmatrix}\) and \(\begin{bmatrix}
1 & 0 \\
3 & 2
\end{bmatrix}\) under the Euclidean inner product on \(M_{22}\).
(b) Let \(V\) be an inner product space. Prove that if \(u\) and \(v\) are orthogonal unit vectors of \(V\) then \(\|u - v\| = \sqrt{2}\).
(c) Let \(R^3\) have Euclidean inner product. Transform the basis \(S = \{u_1, u_2, u_3\}\) into an orthonormal basis using Gram Schmidt process, where \(u_1 = (1, 0, 0), u_2 = (3, 7, -2)\) and \(u_3 = (0, 4, 1)\).

Q.6  (a) Check whether \(T: R^3 \rightarrow R^3\) defined by \(T(x, y, z) = (x + 3y, y, 2x + z)\) is linear. Is it one to one and onto?
(b) Let \(T: R^2 \rightarrow R^3\) be a linear transformation defined by
\(T(x, y) = (y, -5x + 13y, -7x + 16y)\). Find the matrix for \(T\) with respect to the bases \(B_1 = \{u_1, u_2\}\) for \(R^2\) and \(B_2 = \{v_1, v_2, v_3\}\) for \(R^3\), where \(u_1 = (3, 1)\), \(u_2 = (5, 2)\), \(v_1 = (1, 0, -1)\), \(v_2 = (-1, 2, 2)\) and \(v_3 = (0, 1, 2)\).

Q.7  (a) Find the directional derivative of \(xy^2 + yz^3\) at the point \((2, -1, 1)\).
(b) If \(\vec{F} = (y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k\) then show that \(\vec{F}\) is both solenoidal and irrotational.
(c) Verify Green’s theorem for \(\oint_C (3x - 8y^2)dx + (4y - 6xy)dy\) where \(C\) is the boundary of the triangle with vertices \((0, 0)\), \((1, 0)\) and \((0, 1)\).