

**GUJARAT TECHNOLOGICAL UNIVERSITY****BE - SEMESTER- III EXAMINATION – SUMMER 2015****Subject Code:130002****Date:06/06/2015****Subject Name: Advanced Engineering Mathematics****Time: 02.30pm-05.30pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** (a) Attempt the following
- (i) Solve  $[1 + e^{x/y}]dx + e^{x/y}[1 - \frac{x}{y}]dy = 0$  04
- (ii) Solve  $\frac{dy}{dx} + \frac{y}{x^2} = 6e^{1/x}$  03
- (b) Attempt the following
- (i) Solve  $(D^3 - 7D + 6)y = e^{2x}$  04
- (ii) Define square wave function and draw its graph 03
- Q.2** (a) Attempt the following
- (i) Solve  $(D^2 + 9)y = \cos 2x + \sin 2x$  04
- (ii) Find the ordinary and singular points of  $2x^2y'' + 6xy' + (x+3)y = 0$  03
- (b) Attempt the following
- (i) Solve the Cauchy-Euler equation  $x^2D^2y - 3xDy + 5y = x^2 \sin(\log x)$  05
- (ii) Define Gamma Function and obtain its value for 7. 02
- OR**
- (b) Find the series solution of  $(1 + x^2)y'' + xy' - 9y = 0$  07
- Q.3** (a) Find Fourier series for  $f(x) = \begin{cases} \pi x, 0 \leq x \leq 1 \\ \pi(2-x), 1 \leq x \leq 2 \end{cases}$  07
- (b) Attempt the following
- (i) Find Fourier series expansion of  $f(x) = |x|, -\pi < x < \pi$  04
- (ii) Find Fourier sine series for  $f(x) = \pi x - x^2$  in  $(0, \pi)$ . 03
- OR**
- Q.3** (a) Attempt the following
- (i) Obtain Fourier series for  $f(x) = x - x^2, -1 < x < 1$ . 04
- (ii) Find a cosine series for  $f(x) = e^x, 0 < x < \pi$ . 03
- (b) Obtain Fourier series to represent  $f(x) = \left(\frac{\pi - x}{2}\right)^2$  in the interval  $0 < x < 2\pi$ . 07
- Q.4** (a) Attempt the following
- (i) Find the inverse Laplace transform of  $\frac{4s + 5}{(s-1)^2(s+2)}$  04
- (ii) Find the Laplace transform of  $e^{4t} \sin 2t \cos t$  03
- (b) Attempt the following
- (i) Solve by Laplace transform  $y'' + 6y = 1, y(0) = 2, y'(0) = 0$ . 05

(ii) Find the convolution of  $1*1$  02

**OR**

**Q.4 (a)** Attempt the following

(i) Find the inverse Laplace transform of  $\frac{2-5s}{(s-6)(s^2+11)}$  04

(ii) Find Laplace transform of  $t^2 \cosh 3t$ . 03

(b) Attempt the following

(i) Solve by Laplace transform  $y' - 4y = 2e^{2t} + e^{4t}$  given that at  $t = 0, y = 0$ . 05

(ii) Find Laplace transform of  $(t-1)^2 u(t-1)$ . 02

**Q.5 (a)** Attempt the following

(i) Derive partial differential equation by eliminating constants  $a$  and  $b$  from  $z = (x+a)(y+b)$ . 03

(ii) Solve by separation of variables method:  $u_x + u_y = 2(x+y)u$ . 04

(b) Use Frobenius method to solve  $2x^2 y'' - xy' + (1-x^2)y = 0$ . 07

**OR**

**Q.5 (a)** Attempt the following

(i) Form the partial differential equation by eliminating the arbitrary functions  $f$  and  $F$  from the relation  $y = f(x-at) + F(x+at)$ . 03

(ii) Find the complete integral of  $pq = 4z$ . 04

(b) Express the function  $f(x) = \begin{cases} 1 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| \geq 1 \end{cases}$  as a Fourier integral. Hence evaluate

$$\int_0^{\infty} \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda.$$

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