

**GUJARAT TECHNOLOGICAL UNIVERSITY**  
**BE - SEMESTER-III EXAMINATION – WINTER 2015**

**Subject Code:130002****Date:31/12/2015****Subject Name: Advanced Engineering Mathematics****Time: 2:30pm to 5:30pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** (a) Attempt the following
- (i) Solve  $\frac{dy}{dx} + 2y \tan x = \sin x$ . 4
- (ii) Solve  $(e^y + 1) \cos x dx + e^y \sin x dy = 0$ . 3
- (b) Attempt the following
- (i) Solve  $(D^2 - 5D + 6)y = x + e^{4x}$ . 4
- (ii) Define triangular wave function and draw its graph. 3
- Q.2** (a) Attempt the following
- (i) Solve  $(D^2 - 8D + 9)y = 40 \sin 5x$ . 4
- (ii) Find ordinary and singular points for  $2x(x-2)^2 y'' + 3xy' + (x-2)y = 0$ . 3
- (b) Attempt the following
- (i) Solve  $(1+x)^2 y'' + (1+x)y' + y = 4 \cos \log(1+x)$ . 5
- (ii) Define beta function. Find  $B(5,4)$ . 2
- OR**
- (b) Find the power series solution about  $x=0$  of  $y'' + xy' + x^2 y = 0$ . 07
- Q.3** (a) Express  $f(x) = x \cos x$  as a Fourier series in  $(-\pi, \pi)$ . 07
- (b) Find Fourier series expansion of the function given by 07
- $$f(x) = \begin{cases} 0, & -2 < x < 0 \\ 1, & 0 < x < 2 \end{cases}$$
- OR**
- Q.3** (a) If  $f(x) = \begin{cases} x, & 0 \leq x \leq \pi \\ 2\pi - x, & \pi \leq x \leq 2\pi \end{cases}$ , find the Fourier series. 07
- (b) Prove that  $\frac{l}{2} - x = \frac{l}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{2n\pi x}{l}, 0 < x < l$ . 07
- Q.4** (a) Attempt the following
- (i) Evaluate  $L^{-1} \left\{ \log \left( \frac{s+4}{s+3} \right) \right\}$ . 4
- (ii) Find  $L \{ t^2 \sin 4t \}$ . 3
- (b) Attempt the following
- (i) Find the Laplace transform of the periodic function defined by 4
- $$f(t) = \frac{t}{2}, 0 < t < 3, \quad f(t+3) = f(t).$$

- (ii) Find Laplace inverse of  $\frac{1}{s(s+a)^3}$ . 3

**OR**

- Q.4** (a) Attempt the following
- (i) Use convolution theorem to find  $L^{-1}\left\{\frac{1}{s(s^2+a^2)}\right\}$ . 4

- (ii) Find  $L\left\{\frac{1-\cos 2t}{t}\right\}$ . 3

(b) Attempt the following

- (i) Solve by Laplace transform:  $\frac{dy}{dt} - 2y = 4$ , given that  $t = 0, y = 1$ . 5

- (ii) Find  $L(t^2 * \cos t)$ . 2

**Q.5** (a) Attempt the following

- (i) Derive partial differential equation by eliminating  $a$  and  $b$  from  $z = ax + by + ab$ . 3

- (ii) Find the complete integral of  $q = pq + p^2$ . 4

- (b) Solve the p.d.e.  $u_{xx} = 16u_y$  by using separation of variables method. 07

**OR**

**Q.5** (a) Attempt the following

- (i) Form a partial differential equation by eliminating the arbitrary function  $\phi$  from  $\phi(x + y + z, x^2 + y^2 - z^2) = 0$ . 3

- (ii) Solve  $2r + 5s + 2t = 0$ . 4

- (b) Using Fourier integral show that 07

$$\int_0^{\infty} \frac{1 - \cos \pi \lambda}{\lambda} \sin x \lambda d\lambda = \begin{cases} \pi/2, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$$

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