

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER-III EXAMINATION - SUMMER 2016

Subject Code:130002

Date:07/06/2016

Subject Name:Advanced Engineering Mathematics

Time:10:30 AM to 01:30 PM

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** (a) (i) Solve $xy \frac{dy}{dx} = 1 + x + y + xy$ 07
(ii) Solve $\frac{dy}{dx} + \frac{1}{x} = \frac{e^y}{x^2}$
(b) Find the power series solution about $x=0$ of $y'' = 2y'$ 07

- Q.2** (a) Using the method of separation of variables solve 07
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

(b) (i) Solve $(D^3 + 4D)y = \sin 2x$. 07
(ii) Solve $(D^2 + D)y = \frac{1}{1+e^x}$

OR

- (b) (i) Solve $(D^2 - 6D + 9)y = x^2 e^{3x}$. 07
(ii) Solve $(D^2 - 4)y = x^2$
Q.3 (a) Find Fourier series for 07
$$f(x) = \pi x, 0 \leq x \leq 1$$

$$= \pi(2 - x), 1 \leq x \leq 2.$$

(b) Obtain Fourier series for $f(x) = e^{-x}$ in interval $0 < x < 2\pi$. 07

OR

- Q.3** (a) Find Fourier series for 07
$$f(x) = 1 + \frac{2x}{\pi}, -\pi \leq x \leq 0$$

$$= 1 - \frac{2x}{\pi}, 0 \leq x \leq \pi.$$

(b) Obtain half range sine Fourier series for $f(x) = e^x$ in interval $0 < x < 1$ 07

- Q.4** (a) Using Laplace transform solve the differential equation 07
 $\frac{d^2 x}{dt^2} + 2 \frac{dx}{dt} + 5x = e^{-t} \sin t$. where $x(0) = 0, x'(0) = 1$
Using convolution theorem find $L^{-1} \left(\frac{1}{(s^2 + a^2)^2} \right)$.
(b) Using convolution theorem find $L^{-1} \left(\frac{1}{(s^2 + a^2)^2} \right)$ 07

OR

- Q.4** (a) i) Find $L \left(\frac{t - \sin 5t}{t} \right)$. 07
(ii) Find $L (t^2 \cos^2 2t)$.

(b) (i) Find $L^{-1}\left(\frac{1-3s}{s^2+8s+21}\right)$.

07

(ii) Find $L^{-1}\left\{\log\left(\frac{s+a}{s+b}\right)\right\}$

Q.5 (a) (i) Define (1) Heaviside's unit step function (2) Signum function.

07

(ii) Form partial differential equation for $z = f(ax + y) + g(ax - y)$

(b) Find the fourier transform of $f(x) = \frac{1}{x}$

07

OR

Q.5 (a) (i) Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$

07

(ii) Find the complete integral of $p^2 = q + x$

(b) (i) Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$ given that when $x = 0, z = e^y$ and $\frac{\partial z}{\partial x} = 1$

07

(ii) Using Charpit's method solve $z = px + qy + p^2 + q^2$