Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER–III (New) EXAMINATION – WINTER 2015

Subject Code:2130002 Date:31/12/2015
Subject Name: Advanced Engineering Mathematics
Time: 2:30pm to 5:30pm Total Marks: 70

Instructions:
1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **Answer the following one mark each questions:** **14**

1 Find $\Gamma\left(\frac{13}{2}\right)$

2 State relationship between beta and gamma functions.

3 Represent graphically the given saw-tooth function $f(x) = 2x, \quad 0 \le x < 2$ and $f(x+2) = f(x)$ for all x.

4 For a periodic function f with fundamental period p, state the formula to find Laplace transform of f.

5 Find $L\left(e^{-3t}f(t)\right)$, if $L\left(f(t)\right) = \frac{s}{(s-3)^2}$.

6 Find $L[(2t-1)^2]$.

7 Find the extension of the function $f(x) = x + 1$, define over $(0,1]$ to $[-1,1] - \{0\}$ which is an odd function.

8 Is the function $f(x) = \begin{cases} x, & 0 \le x \le 2 \\ x^2, & 2 < x \le 4 \end{cases}$; continuous on $[0,4]$? Give reason.

9 Is the differential equation $\frac{dy}{dx} = \frac{y}{x}$ exact? Give reason.

10 Give the differential equation of the orthogonal trajectory to the equation $y = cx^2$.

11 If $y = c_1y_1 + c_2y_2 = e^x(c_1 \cos x + c_2 \sin x)$ is a complementary function of a second order differential equation, find the Wronskian $W(y_1, y_2)$.

12 Solve $(D^2 + D + 1)y = 0$; where $D = \frac{d}{dt}$.

13 Is $u(t,x) = 50e^{(t-x)/2}$, a solution to $\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} + u$?

14 Give an example of a first order partial differential equation of Clairaut's form.

Q.2 **(a)** Solve: $\frac{dy}{dx} = \frac{x^2-x-y^2}{2xy}$. **03**

	(b)	Solve: $\frac{dy}{dx}+\frac{1}{x}y=x^3y^3$.	**04**
	(c)	Find the series solution of $(x-2)\frac{d^2y}{dx^2}-x^2\frac{dy}{dx}+9y=0$ about $x_0=0$.	**07**
		OR	
	(c)	Explain regular-singular point of a second order differential equation and find the roots of the indicial equation to $x^2y''+xy'-(2-x)y=0$.	**07**
Q.3	**(a)**	Find the complete solution of $\frac{d^3y}{dx^3}+8y=\cosh(2x)$.	**03**
	(b)	Find solution of $\frac{d^2y}{dx^2}+9y=\tan 3x$, using the method of variation of parameters.	**04**
	(c)	Using separable variable technique find the acceptable general solution to the one-dimensional heat equation $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$ and find the solution satisfying the conditions $u(0,t)=u(\pi,t)=0$ for $t>0$ and $u(x,0)=\pi-x,\quad 0<x<\pi$.	**07**
		OR	
Q.3	**(a)**	Solve completely, the differential equation $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=\cos(2x)\sin x$.	**03**
	(b)	Solve completely the differential equation $x^2\frac{d^2y}{dx^2}-6x\frac{dy}{dx}+6y=x^{-3}\log x$.	**04**
	(c)	(i) Form the partial differential equation for the equation $(x-a)(y-b)-z^2=x^2+y^2$. (ii) Find the general solution to the partial differential equation $xp+yq=x-y$.	**07**
Q.4	**(a)**	Find the Fourier cosine integral of $f(x)=\frac{\pi}{2}e^{-x}, x\geq 0$.	**03**
	(b)	For the function $f(x)=\cos 2x$, find its Fourier sine series over $[0,\pi]$.	**04**
	(c)	For the function $f(x)=\begin{cases} x; & 0\leq x\leq 2 \\ 4-x; & 2\leq x\leq 4 \end{cases}$, find its Fourier series. Hence show that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots=\frac{\pi^2}{16}$.	**07**
		OR	
Q.4	**(a)**	Find the Fourier cosine series of $f(x)=e^{-x}$, where $0\leq x\leq\pi$.	**03**
	(b)	Show that $\int_0^\infty \frac{\lambda^3\sin\lambda x}{\lambda^4+4}d\lambda=\frac{\pi}{2}e^{-x}\cos x, x>0$.	**04**
	(c)	Is the function $f(x)=x+\lvert x\rvert$, $-\pi\leq x\leq\pi$ even or odd? Find its Fourier series over the interval mentioned.	**07**
Q.5	**(a)**	Find $L\left\{\int_0^t e^u(u+\sin u)du\right\}$.	**03**
	(b)	Find $L^{-1}\left\{\frac{1}{s(s^2-3s+3)}\right\}$.	**04**
	(c)	Solve the initial value problem: $y''-2y'=e^t\sin t, y(0)=y'(0)=0$, using Laplace transform.	**07**
		OR	
Q.5	**(a)**	Find $L\{t(\sin t-t\cos t)\}$.	**03**
	(b)	Find $L^{-1}\left\{\frac{e^{-2s}}{(s^2+2)(s^2-3)}\right\}$.	**04**
	(c)	State the convolution theorem and verify it for $f(t)=t$ and $g(t)=e^{2t}$.	**07**

****