Seat No.: ________ Enrolment No.___________

# GUJARAT TECHNOLOGICAL UNIVERSITY

**BE - SEMESTER–III(New) • EXAMINATION – WINTER 2016**

**Subject Code:2130002** **Date:30/12/2016**

**Subject Name:Advanced Engineering Mathematics**

**Time:10:30 AM to 01:30 PM** **Total Marks: 70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

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|---|---|---|---|---|
| | | | | **MARKS** |
| **Q.1** | | **Answer the following one mark questions** | | **14** |
| | 1 | Find $\Gamma\left(\frac{1}{2}\right)$. | | |
| | 2 | State relation between beta and gamma function. | | |
| | 3 | Define Heaviside's unit step function. | | |
| | 4 | Define Laplace transform of f (t), t $\geq$ 0. | | |
| | 5 | Find Laplace transform of $t^{\frac{-1}{2}}$. | | |
| | 6 | Find L { $\frac{sinat}{t}$ }, given that L { $\frac{sint}{t}$ } = $tan^{-1}\{\frac{1}{s}\}$. | | |
| | 7 | Find the continuous extension of the function f(x) = $\frac{x^2+x-2}{x^2-1}$ to x = 1 | | |
| | 8 | Is the function f(x) = $\frac{1}{x}$ continuous on [-1, 1] ? Give reason. | | |
| | 9 | Solve $\frac{dy}{dx} = e^{3x-2y} + x^2e^{-2y}$. | | |
| | 10 | Give the differential equation of the orthogonal trajectory of the family of circles $x^2 + y^2 = a^2$. | | |
| | 11 | Find the Wronskian of the two function sin2x and cos2x . | | |
| | 12 | Solve ( $D^2 + 6D + 9$) x = 0; D = $\frac{d}{dt}$. | | |
| | 13 | To solve heat equation $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$ how many initial and boundary conditions are required. | | |
| | 14 | Form the partial differential equations from z = f(x + at) + g(x –at). | | |
| **Q.2** | **(a)** | Solve : (x+1)$\frac{dy}{dx} - y = e^{3x}(x+1)^2$. | | **03** |
| | **(b)** | Solve : $\frac{dy}{dx} + \frac{ycosx + siny + y}{sinx + xcosy + x} = 0$ | | **04** |
| | **(c)** | Find the series solution of $\frac{d^2y}{dx^2} + xy = 0$. | | **07** |
| | | **OR** | | |
| | **(c)** | Find the general solution of $2x^2y'' + xy' + (x^2 - 1)y = 0$ by using frobenius method. | | **07** |
| **Q.3** | **(a)** | Solve : $(D^3 - 3D^2 + 9D - 27)$y = cos3x. | | **03** |
| | **(b)** | Solve : $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = x^2\sin(lnx)$. | | **04** |
| | **(c)** | (i) Solve : $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2\partial y} = 2e^{2x}$. | | **03** |
| | | (ii) find the general solution to the partial differential equation | | **04** |

$(x^2 - y^2 - z^2)p + 2xyq = 2xz.$

**OR**

**Q.3** **(a)** Solve : $(D^3 - D)y = x^3$ . **03**

**(b)** Find the solution of $y'' - 3y' + 2y = e^x$ , using the method of variation of parameters. **04**

**(c)** Solve $x\frac{\partial u}{\partial x}$ - 2y $\frac{\partial u}{\partial y} = 0$ using method of separation of variables. **07**

**Q.4** **(a)** Find the Fourier cosine integral of $f(x) = e^{-kx}, x > 0, k > 0$ **03**

**(b)** Express f(x) = |x|, $-\pi < x < \pi$ as fouries series. **04**

**(c)** Find Fourier Series for the function f(x) given by **07**

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi};\ -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}\ ; 0 \le x \le \pi \end{cases}$$

Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots . = \frac{\pi^2}{8}$ .

**OR**

**Q.4** **(a)** Obtain the Fourier Series of periodic function function f(x) =2x, -1 < x < 1, p = 2L =2 **03**

**(b)** Show that $\int_0^\infty \frac{sin\lambda cos\lambda}{\lambda} d\lambda$ = 0, if x > 1. **04**

**(c)** Expand f(x) in Fourier series in the interval $(0, 2\pi)$ if **07**

$$f(x) = \begin{cases} -\pi\ ; 0 < x < \pi \\ x - \pi\ ; \pi < x < 2\pi \end{cases}$$

and hence show that $\sum_{r=0}^{\infty} \frac{1}{(2r+1)^2} = \frac{\pi^2}{8}$.

**Q.5** **(a)** Find L$\{\int_0^t e^t \frac{sint}{t} dt\}$. **03**

**(b)** Find $L^{-1}\{\frac{2s^2-1}{(s^2+1)(s^2+4)}\}$. **04**

**(c)** Solve initial value problem : $y'' - 3y'$ + 2y = 4t + $e^{3t}$ ,y(0) = 1 and $y'(0) = -1$ ,using Laplace transform. **07**

**OR**

**Q.5** **(a)** Find L {tsin3tcos2t}. **03**

**(b)** Find $L^{-1}\{\frac{e^{-3s}}{s^2+8s+25}\}$. **04**

**(c)** State the convolution theorem and apply it to evaluate $L^{-1}\{\frac{s}{(s^2+a^2)^2}\}$. **07**

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